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GA—104—2023

FACULTY OF SCIENCE

B.Sc. (Fourth Semester) EXAMINATION

APRIL/MAY, 2023

(New Pattern)

MATHEMATICS

Paper X

(Ring Theory)

(Tuesday, 9-5-2023) **Time : 2.00 p.m. to 4.00 p.m.**

Time—Two Hours *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Define ring, commutative ring and prove that every field is an integral domain. 15

Or

(*a*) Show that the characteristic of an integral domain is either *o* or *a* prime number. 8

(*b*) If D is an integral domain then show that the polynomial ring D[*x*] is also an integral domain. 7

P.T.O.

2. Let $f(x), g(x) \neq 0$ be any two polynomials of the polynomial domain F[x] over the field F. Then there exist uniquely two polynomials $q(x)$ and $r(x)$ in F[x] such that : 15

$$f(x) = q(x)\, g(x) + r(x)$$

where either :

$$r(x) = 0 \text{ or } \deg r(x) < \deg g(x)$$

Or

(*a*) Show that a commutative ring with unity is a field if it has no proper ideals. 8

(*b*) Every Euclidean ring possesses unity element. 7

3. Attempt any *two* of the following : 10

(*a*) Let R be a commutative ring and S an ideal of R. Then the ring of residue classes R|S is an integral domain if and only if S is a prime ideal.

(*b*) Add and multiply the following polynomials over the ring of integers :

$$f(x) = 2x^0 + 5x + 3x^2 - 4x^3$$

$$g(x) = 3x^0 + 4x - x^3 + 5x^4$$

(*c*) Show that the set of matrices :

$$\begin{bmatrix} a & b \\ o & c \end{bmatrix}$$

is a subring of the ring 2 × 2 matrices with an integral elements.

(*d*) Prove that if $a, b \in \mathrm{R}$, then :

$$(a + b)^2 = a^2 + ab + ba + b^2$$

where by x^2, we mean x^x.