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GA—97—2023

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Second Year) (Third Semester) EXAMINATION
APRIL/MAY, 2023

(New Pattern)

MATHEMATICS

Paper VII

(Group Theory)

(Monday, 8-5-2023) **Time : 2.00 p.m. to 4.00 p.m.**

Time— Two Hours *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Prove that inverse of each element of a group is unique, also show that $(ab)^{-1} = b^{-1} a^{-1} \ \forall \ a, b \in G$ where (G .) is a group. 15

Or

(*a*) Prove that the orders of the element a and $x^{-1} ax$ are same, where a, x are any two elements of a group ? 8

(*b*) Prove that if G is an abelian group, then for all $a, b \in G$ and all integers n, $(ab)^n = a^n b^n$. 7

2. State and prove Cayley's theorem. 15

Or

(*a*) Prove that every Homomorphic image of a group G is isomorphic to some quotient group of G. 8

(*b*) If H is a subgroup of G and M is a normal subgroup of G, then, show that $H \cap M$ is a normal subgroup of H. 7

P.T.O.

3. Attempt any *two* of the following : 5 each

(*a*) If $f = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

be two permutation of degree 3, then show that

$$fg = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \text{ and } gf = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$

(*b*) Show that the set I of all integers is a group with respect to the operation of addition of integers.

(*c*) Prove that every cyclic group is abelian group.

(*d*) Show that $a \to a^{-1}$ is an automorphism of a group G iff G is abelian.