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GA—91—2023

FACULTY OF SCIENCE AND TECHNOLOGY

B.Sc. (First Year) (First Semester) EXAMINATION

APRIL/MAY, 2023

(New Course)

MATHEMATICS

Paper II

(Algebra and Trigonometry)

(Monday, 8-5-2023) **Time : 10.00 a.m. to 12.00 noon**

Time— Two Hours *Maximum Marks*—40

N.B. :— (*i*) Attempt *All* questions.

(*ii*) Figures to the right indicate full marks.

1. If A be an-square matrix, then prove that : 15

$$A(\text{adj} . A) = (\text{adj} . A) A = |A| \, I_n$$

where In denotes the unit matrix of order n and verify the above result for a matrix :

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}.$$

Or

(*a*) Prove that a system AX = B of m linear equations in n unknowns is consistent if and only if the coefficient matrix A and the augmented matrix [A : B] of the system have the same rank.

P.T.O.

Also, examine the consistency of the following equations : 8

$$x + y + z = 6$$

$$x + 2y + 3z = 14$$

$$x + 4y + 7z = 30.$$

(*b*) Prove that λ is characteristic root of a matrix A if and only if there exists a non-zero vector X, such that : 7

$$AX = \lambda X.$$

2. State and prove De Moivre's theorem and simplify : 15

$$\frac{(\cos 3\theta + i \sin 3\theta)^5 (\cos \theta - i \sin \theta)^3}{(\cos 5\theta + i \sin 5\theta)^7 (\cos 2\theta - i \sin 2\theta)^5}.$$

Or

(*a*) Prove that the elementary operations do not alter the rank of a matrix. 8

(*b*) Find a row echelon matrix, which is row equivalent to ? 7

$$A = \begin{bmatrix} 0 & 0 & -2 & 3 & 1 \\ 2 & 4 & 1 & 4 & 3 \\ 1 & 2 & -3 & 1 & 2 \\ 4 & 8 & 2 & 3 & 5 \end{bmatrix}$$

and find ρ(A) R.

3. Attempt any *two* of the following : 5 each

(*a*) Prove that the product of matrices is not commutative in general. i.e., prove AB ≠ BA discussing all possibilities.

(*b*) Define the row rank and reduced the following matrix to row echelon matrix :

$$A = \begin{bmatrix} 1 & -2 & -1 & 4 \\ 2 & -4 & 3 & 5 \\ -1 & 2 & 6 & -7 \end{bmatrix}.$$

(*c*) Solve the equations :

$$x + 2y + 3z + 4t = 0$$
$$8x + 5y + z + 4t = 0$$
$$5x + 6y + 8z + t = 0$$
$$8x + 3y + 7z + 2t = 0.$$

(*d*) If $x = \cos\theta + i\sin\theta$, then prove that :

$$x^n + \frac{1}{x^n} = 2\cos n\theta$$

$$x^n - \frac{1}{x^n} = 2i\sin n\theta.$$