This question paper contains **3** printed pages]

GA—89—2023

FACULTY OF SCIENCE/ARTS

B.Sc. (Second Year) (Fourth Semester) EXAMINATION

APRIL/MAY, 2023

(New Course)

MATHEMATICS

Paper IX

(Real Analysis-II)

(Saturday, 6-5-2023) **Time : 2.00 p.m. to 4.00 p.m.**

Time— Two Hours *Maximum Marks*—40

N.B. :— (*i*) Attempt *All* questions.

(*ii*) Figures to the right indicate full marks.

1. Prove that the oscillation of a bounded function of on an interval $[a, b]$ is the supremum of the set :

$$\{|f(x_1) - f(x_2)| : x_1, x_2 \in [a, b]\}$$

of numbers. Also, show that the function f defined by :

$$f(x) = \begin{cases} 0, \text{ when } x \text{ is rational} \\ 1, \text{ when } x \text{ is irrational} \end{cases}$$

is not integrable on any interval. 15

Or

(*a*) State and prove :

(*i*) First Mean Value Theorem

(*ii*) Generalised First Mean Value Theorem. 8

(*b*) Compute $\int_{-1}^{1} f(x)\,dx$, where $f(x) = |x|$. 7

P.T.O.

2. State and prove : 15

(*i*) Comparison Test I

(*ii*) Comparison Test II.

Or

(*a*) If ϕ is bounded and monotonic in $[a, \infty[$ and tends to 0 as $x \to \infty$ and $\int_a^X f\, dx$ is bounded for $X \geq a$, then prove that $\int_a^\infty f\, \phi\, dx$ is convergent at ∞. 8

(*b*) Show that the integral $\int_a^\infty x^{m-1} e^{-x}\, dx$ is convergent if and only if $m > 0$. 7

3. Attempt any *two* of the following :

(*a*) If f_1, f_2 are two bounded and integrable functions on $[a, b]$, then prove that $f = f_1 - f_2$ is also integrable on $[a, b]$. 5

(*b*) If f and g are integrable in $[a, b]$ and g keeps the same sign over $[a, b]$, then prove that there exists a number μ lying between the bounds of f such that : 5

$$\int_a^b f\ g\ dx = \mu \int_a^b g\ dx.$$

(*c*) Show that the integral : 5

$$\int_0^{\pi/2} \left(\frac{\sin^m x}{x^n} \right) dx$$

exists if and only if $n < m + 1$.

(*d*) Show that : 5

$$\int_1^{\infty} \frac{\sin x}{x^p} dx$$

converges absolutely if $p > 1$.