This question paper contains **3** printed pages]

GA—80—2023

FACULTY OF ARTS/SCIENCE

B.Sc. (Second Year) (Third Semester) EXAMINATION

APRIL/MAY, 2023

(New Pattern)

MATHEMATICS

Paper VI

(Real Analysis-I)

(Thursday, 4-5-2023) **Time : 2.00 p.m. to 4.00 p.m.**

Time— Two Hours *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Attemp the following :

(*a*) A set is closed if and only if its complement is open. 8

(*b*) Show that $\sqrt{8}$ is not a rational number. 7

Or

Attempt the following :

(*a*) Prove that, a necessary and sufficient condition for the convergence of the sequence $\{S_n\}$ is that, for each $\in > 0$ there exists a positive integer m such that : 8

$$|S_{n+p} - S_n| < \varepsilon, \forall\ n \geq m \wedge p \geq 1.$$

(*b*) Show that : 7

$$\lim_{n\to\infty} \frac{3 + 2\sqrt{n}}{\sqrt{n}} = 2.$$

P.T.O.

2. (A) Attempt the following

(*i*) If $\{a_n\}$, $\{b_n\}$ are two sequences such that : 8

(1) $a_n \leq b_n, \ \forall \ n$ and

(2) $\lim_{n\to\infty} a_n = a, \ \lim_{n\to\infty} b_n = b$

then prove that $a \leq b$.

(*ii*) Show that the sequence $\{S_n\}$, where $S_n = \left(1+\frac{1}{n}\right)^n$, is convergent and that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ lies between 2 and 3. 7

Or

(B) Attempt the following : 8

(*i*) If Σu_n is a positive term series such that $\lim_{n\to\infty}(u_n)^{1/n} = l$, then prove that the series :

(1) Converges is $l < 1$

(2) Diverges if $l > 1$

(3) The test fails to give any definite information, if $l = 1$.

(*ii*) Show that the series : 7

$$1+\frac{1}{2\,!}+\frac{1}{3\,!}+\frac{1}{4\,!}+\ldots\ldots+\ldots..$$

is convergent.

3. Attempt any *two* of the following : 10

(*a*) Write the properties for a set 'S' to be a field structure.

(*b*) If $\{a_n\}$ and $\{b_n\}$ be the two sequences such that :

$$\lim_{n\to\infty} a_n = a, \lim_{n\to\infty} b_n = b$$

then prove that :

$$\lim_{n\to\infty} (a_n - b_n) = \lim_{n\to\infty} a_n - \lim_{n\to\infty} b_n = a - b.$$

(*c*) Show that the series $\Sigma \frac{1}{n}$ does not converge.

(*d*) Show that the series :

$$\frac{1}{(\log 2)^p} + \frac{1}{(\log 3)^p} + \frac{1}{(\log 4)^p} + ...+ \frac{1}{(\log n)^p} + ...$$

diverges for $p > 0$.