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GA—74/77—2023

FACULTY OF SCIENCE AND TECHNOLOGY

B.Sc. (Third Year) (Sixth Semester) EXAMINATION

APRIL/MAY, 2023

(CBCS/New)

MATHEMATICS

Paper-XVII

(Topology)

(Thursday, 4-5-2023) **Time : 10.00 a.m. to 12.00 noon**

Time— Two Hours *Maximum Marks*—40

N.B. :— (*i*) Attempt either A or B for question Nos. **1** and **2**.

(*ii*) All symbols carry equal marks.

(*iii*) Figures to the right indicate full marks.

1. (A) Attempt the following :

(*i*) Let X be topological space. Suppose that **C** is a collection of open sets of X such that for each open set U of X and each x in U. There is an element C of **C** such that $x \in C \subset U$. Then show that C is a basis of the topology on X. 8

(*ii*) If X is any set, then show that the collection of all one-point subsets of X is a basis for the discrete topology on X. 7

Or

(B) Attempt the following :

(*i*) If A is a subspace of X and B is a subspace of Y, then prove that the product topology on $A \times B$ is same as the topology $A \times B$ inherits as a subspace of $X \times Y$. 7

P.T.O.

(*ii*) Define projection maps and prove that projection maps are open maps. 8

2. (A) Attempt the following :

(*i*) Let A be a subset of the topological space X. Then prove that : 8

(*a*) $x \in \overline{A}$ iff every open set U containing x intersect A.

(*b*) Supposing the topology of X is given by a basis, then $x \in \overline{A}$ iff every basis element B containing x intersect A.

(*ii*) Define Hausdorff space. Hence prove that : 7

Every finite point set in a Hausdorff space X is closed.

Or

(B) Attempt the following :

(*a*) Define continuous functions. Let $X = A \cup B$, where A and B are closed in X. Let $f : A \to Y$ and $g : B \to Y$ be continuous. If $f(x) = g(x)$ for every $x \in A \cap B$, then show that f and g combine to give a continuous function $h : X \to Y$, defined by setting $h(x) = f(x)$ if $x \in A$ and $h(x) = g(x)$ if $x \in B$. 7

(*b*) Define connected sets. If the sets C and D form a separation of X, and if Y is a connected subspace of X, then show that Y lies entirely within C or D. 8

3. Attempt any *two* of the following : 5 each

(*a*) Prove that the collection of all circular region (interiors of circle) is the basis for a topology.

(*b*) Define an open set in subspace topology for Y. Let Y be a subspace of X; If U is open in Y and Y is open in X, then show that U is open in X.

(*c*) Let $A \subset X$ and $B \subset Y$, show that in the space $X \times Y$, $\overline{A \times B} = \overline{A} \times \overline{B}$.

(*d*) Define compact set. Prove that the set $X = \{0\} \cup \left\{\frac{1}{n} / n \in Z_+\right\}$ is compact.