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GA—72—2023

FACULTY OF SCIENCE/ARTS

B.Sc. (First Year) (First Semester) EXAMINATION

APRIL/MAY, 2023

(New Pattern)

MATHEMATICS

Paper I

(Calculus)

(Thursday, 4-5-2023) **Time : 10.00 a.m. to 12.00 noon**

Time— Two Hours *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory..

(*ii*) Figures to the right indicate full marks.

1. Let $F(x + h)$ be a function of h (x being independent of h) which can be expanded in powers of h and the expansion be differentiable any number of times, then prove that : 15

$$F(x + h) = F(x) + hF'(x) + \frac{h^2}{2!} F''(x) + ... + \frac{h^n}{n!} F^{(n)}(x) + ...$$

also expand $\log (x + b)$ in powers of x by Taylor's theorem.

Or

(*a*) If $y = \sin(ax + b)$, then find $\frac{d^n y}{dx^n}$. 8

(*b*) If $y = \sin(\sin x)$, prove that : 7

$$\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y \cos^2 x = 0.$$

P.T.O.

2. If two functions $f(x)$ and $F(x)$ are derivable in a closed interval $[a, b]$ and $F'(x) \neq 0$, for any value of x in $[a, b]$, then there exists at least one value 'c' of x belonging to the open interval $]a, b[$ such that :

$$\frac{F(b) - F(a)}{F(b) - F(a)} = \frac{F'(c)}{F'(c)}$$

Also in the Cauchy's mean value theorem, if $F(x) = e^x$ and $F(x) = e^{-x}$, show that c is arithmetic mean between a and b. 15

Or

(*a*) If $z = F(x, y)$ be a homogeneous function of x, y of degree n, then prove that : 8

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz, \forall\ x, y \in$$

to the domain of the function.

(*b*) If $u = \log(\tan x + \tan y + \tan z)$, then prove that : 7

$$(\sin 2x)\frac{\partial u}{\partial x} + (\sin 2y)\frac{\partial u}{\partial y} + (\sin 2z)\frac{\partial u}{\partial z} = 2.$$

3. Attempt any *two* of the following : 5 each

(*a*) If $y = \cos^4 x$, then find nth derivative of y.

(*b*) Show that in the case of the curve $\beta y^2 = (x + \alpha)^3$, prove that the square of the subtangent varies as the subnormal.

(*c*) Show that :

$$\frac{\sin \alpha - \sin \beta}{\cos \beta - \cos \alpha} = \cot \theta,$$

$$0 < \alpha < \theta < \beta < \frac{\pi}{2}.$$

(*d*) If :

$u = 3(lx + my + nz)^2 - (x^2 + y^2 + z^2)$ and $l^2 + m^2 + n^2 = 1$, then show that :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0.$$