This question paper contains **2** printed pages]

GA—54/55—2023

FACULTY OF SCIENCE

B.Sc. (Third Year) (Sixth Semester) EXAMINATION

APRIL/MAY 2023

(CBCS/New/Old Pattern)

MATHEMATICS

Paper-XVI

(Integral Transforms)

(Tuesday, 2-5-2023) **Time : 10.00 a.m. to 12.00 noon**

Time— Two Hours *Maximum Marks*—40

N.B. :— (*i*) Attempt *all* questions.

(*ii*) Figures to the right indicates full marks.

1. If $L[f(t)] = F(s)$, then prove that $L[t^n f(t)] = (-1)^n \frac{d^n}{d8^n}[F(s)]$ and find the Laplace transform of $t \sinh at$. 15

Or

(*a*) Find the inverse Laplace transform of : 8

(*i*) $\frac{s}{4s^2 - 25}$

(*ii*) $\frac{1}{s(s^2 + 1)}$

(*b*) Find $L^{-1}\left[\frac{1}{s(s^2 + a^2)}\right]$. 7

P.T.O.

2. Prove the Fourier sine integral 15

$$f(x) = \frac{2}{\pi}\int_0^\infty \sin ux\, du . \int_0^\infty f(t) \sin ut\, dt$$

and the Fourier cosine integral

$$f(x) = \frac{2}{\pi}\int_0^\infty \cos ux\, du . \int_0^\infty f(t) \cos ut\, dt .$$

Or

(*a*) Using Laplace transforms find the solution of the initial value problem : 8

$$y'' + 9y = 6 \cos 3t$$
$$y(0) = 2,\ y'(0) = 0$$

(*b*) Solve the initial value problem : 7

$$2y'' + 5y' + 2y = e^{-2t}$$
$$y(0) = 1,\ y'(0) = 1$$

Using Laplace transforms.

3. Attempt any *two* of the following : 5 each

(*a*) Prove that L (sinh at) = $\frac{a}{s^2 - a^2}$.

(*b*) Find inverse Laplace transform of $\tan^{-1}\frac{1}{s}$.

(*c*) Using Laplace transforms find the solution of the initial value problem $y'' - 4y' + 4y = 64 \sin 2t$.

(*d*) If $f(s)$ is the complex Fourier transform of $f(x)$, then prove that

$$F\{f(ax)\} = \frac{1}{a} F\left(\frac{s}{a}\right).$$