This question paper contains 4 printed pages]

WT—07—2024

FACULTY OF SCIENCE

M.Sc. (First Year) (First Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

(CBCS/New Pattern)

PHYSICS

PHY-101

(Mathematical Methods in Physics)

(Tuesday, 10-12-2024) **Time : 10.00 a.m. to 1.00 p.m.**

Time—3 *Hours* *Maximum Marks*—75

N.B. :— (*i*) Attempt *all* questions

(*ii*) *All* questions carry equal marks.

1. Find the eigen values, eigen vectors and diagonal matrix of the following : 15

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

P.T.O.

Or

(*a*) Solve the following system of linear non-homogeneous equation : 8

$$x + y + z = 6$$

$$x - y + z = 2$$

$$2x + y - z = 1$$

(*b*) Find the inverse of the following matrix : 7

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

2. Using Legendre's polynomial, show that :

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2 - 1)^n \text{ and}$$ 15

find the values of $P_0(x)$, $P_1(x)$ and $P_2(x)$.

Or

(*a*) Show that : 8

(*i*) $2xH_n(x) - H_{n+1}(x) = 2nH_{n-1}(x)$

(*ii*) $2nH_{n-1}(x) = H'_n(x)$.

(*b*) Derive an expression for generating function of Bessel polynomial. 7

3. Define the Fourier series and Fourier coefficients and find the Fourier series for the following function : 15

$$f(x) = \pi - x,\ 0 < x < \pi$$

Or

(*a*) Solve the following differential equation using Laplace transform : 8

$$y'' + 25y = 10 \cos 5t,$$

where $y(0) = 2$, $y'(0) = 0$.

(*b*) If F(*s*) is the Fourier transform of F(*x*), then show that : 7

$$F\,[F(x) \cos ax] = \frac{1}{2}[F(s + a) + F(s - a)].$$

4. Show that the sufficient condition for a function $F(z) = u + iv$ to be analytic at all points in the region 'R' are : 15

(*i*) $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$

(*ii*) $\dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$

(*iii*) $\dfrac{\partial u}{\partial x}, \dfrac{\partial u}{\partial y}, \dfrac{\partial v}{\partial x}, \dfrac{\partial v}{\partial y}$ are continuous functions of x and y in the region 'R' and also check whether the following is analytic or not :

$$F(z) = z^3.$$

Or

(*a*) Evaluate $\int_C (x + y)\,dx + x^2 y\,dy$: 8

(*i*) along $y = x^2$ having (0, 0) and (3, 9) as end points

(*ii*) along $y = 3x$ between (0, 0) and (3, 9).

P.T.O.

(*b*) If $f(z)$ is analytic in a closed curve 'C' except at a finite no. of poles within 'C', then show that : 7

$$\int_C F(z)\,dz = 2\pi i\,[\text{sum of residues at the poles in 'C'}]$$

5. Write short notes on (any *three*) : 15

(*a*) Rotation of a matrix

(*b*) Cauchy integral formula

(*c*) Linearity and first shifting properties of Fourier transform

(*d*) Recurrence relations of $J_n(x)$.