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VA—81—2024

FACULTY OF SCIENCE AND TECHNOLOGY

B.Sc. (Second Year) (Fourth Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

(New Pattern)

MATHEMATICS

Paper–X

(Ring Theory)

(Friday, 13-12-2024) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. A ring R is without zero divisors if and only if the cancellation laws hold in R. 15

Also prove that a skew field has no divisors of zero.

Or

(*a*) Define isomorphism of rings. If f is an isomorphism of a ring R onto a ring R', then prove that : 8

(*i*) If R is without zero divisors, then R' is also a without zero divisors.

(*ii*) If R is with unit element, then R' is also with unit element.

P.T.O.

(*b*) Prove that the characteristic of an integral domain is either zero or a prime number. 7

2. Prove that an ideal S of a commutative ring R with unity is matimal ideal if and only if the residue class ring R/S is a field. 15

Or

(*a*) Find out the units of the integral domain of Gaussian integers. 8

(*b*) If D is an integral domain, then the polynomial ring D(x) is also an integral domain. 7

3. Attempt any *two* of the following : 10

(*a*) If a, b, c, d are any elements of a ring R, prove that

$(a - b)(c - d) = (ac + bd) - (ad + bc)$.

(*b*) If R is a ring and $a \in$ R. Let $T = \{x \in R : ax = 0\}$. Prove that T is a right ideal of R.

(*c*) Add and multiply the following polynomials over the ring

$(I_6, +_6, \times_6)$

$$f(x) = 2x^{\circ} + 5x + 3x^2$$

$$g(x) = x^{\circ} + 4x + 2x^2.$$

(*d*) If f is a homomorphism of a ring R into a ring R' with kernel S, then S is an ideal of ring R.