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VA—74—2024

FACULTY OF SCIENCE

B.Sc. (Second Year) (Third Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

(New Pattern)

MATHEMATICS

Paper VII

(Group Theory)

(Thursday, 12-12-2024) **Time : 2.00 p.m. to 4.00 p.m.**

Time—Two Hours *Maximum Marks*—40

N.B. :— (*i*) Attempt *All* questions.

(*ii*) Figures to the right indicate full marks.

1. If $f : X \to Y$ and $g : Y \to Z$ be two one-one and onto maps, then prove that $gof : X \to Z$ is also one-one and onto. Also prove that : 15

$$(gof)^{-1} : Z \to X = (f^{-1} o g^{-1}) : Z \to X$$

Or

(*a*) State and prove Lagrange's theorem. 8

(*b*) If H is a subgroup of the group G, then prove that there is one-to-one correspondence between any two right cosets of H in G. 7

P.T.O.

2. If f is a homomorphism of a group G into a group G', then prove that : 15

(*i*) $f(e) = e'$, where e is the identity of G and e' is the identity of G'.

(*ii*) $f(a^{-1}) = [f(a)]^{-1}$, for all $a \in G$.

(*iii*) If the order of $a \in G$ is finite, then the order of $f(a)$ a divisor of the order of a.

Or

(*a*) Prove that a non-empty subset H of a group G is a subgroup of G if and only if : 8

(*i*) $a \in H$, $b \in H \Rightarrow ab \in H$

(*ii*) $a \in H \Rightarrow a^{-1} \in H$

where a^{-1} is the inverse of a in G.

(*b*) Let $f = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ be two permutations of degree 3, then find fg and gf. Is $fg = gf$? 7

3. Attempt any *two* of the following : 5 each

(*a*) Let I be the set of integers and R be the relation in I defined by, for $x, y \in I$, $x R y$ iff $x - y$ is divisible by 5. Then prove that R is an equivalence relation.

(*b*) Prepare the composition table for the group G = {0, 1, 2, 3, 4} with respect to 'addition modulo 5' and write down the inverse of each element of G.

(*c*) Prove that every cyclic group is an abelian group.

(*d*) Let G be a group and let e be the identity element of G. Then prove that the mapping $f : G \to G$ defined by $f(a) = e$, for all $a \in G$ is a homomorphism of G into G.