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VA—58—2024

FACULTY OF SCIENCE

B.Sc. (Third Year) (Sixth Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

(CBCS/New Pattern)

MATHEMATICS

Paper–XVII

(Elementary Number Theory)

(Tuesday, 10-12-2024) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Prove that the linear Diophantine equation $ax + by = c$ has a solution if and only if $d|c$ where $d = gcd\ (a, b)$. If x_0, y_0 is any particular solution of this equation, then all other solutions are given by $x = x_0 + \left(\frac{b}{d}\right)t, y = y_0 - \left(\frac{a}{d}\right)t$ where t is any arbitrary integer. 15

Or

(*a*) If $a = qb + r$, $k > 0$, then prove that $gcd\ (a, b) = gcd(b, r)$ and $gcd(ka, kb) = kgcd(a, b)$. 8

P.T.O.

(*b*) By using Mathematical induction establish the result : 7

$$1^2 + 2^2 + 3^2 + \ldots \ldots\ldots \ldots + n^2 = \frac{n(2n+1)(n+1)}{6}$$

2. Let $n_1, n_2, \ldots\ldots, n_r$ be positive integers such that $gcd\ (n_i, n_j) = 1$ for $i \neq j$. Prove that the system of linear congruences $x \equiv a_1\ (mod\ n_1)$, $x \equiv a_2\ (mod\ n_2)$, ..., $x \equiv a_r\ (mod\ n_r)$ has a simultaneous solutions which is unique modulo integers $n_1, n_2 \ldots\ldots\ldots n_r$. 15

Or

(*a*) If all the $n > 2$, terms of the arithmetic progression $p, p + d, p + 2d$, ... $p + (n - 1)\,d$ are prime numbers, show that the common difference d is divisible by every prime $q < n$. 8

(*b*) Find the remainder obtained upon dividing the sum : 7

$1! + 2! + 3! + \ldots\ldots\ldots \ldots\ldots \ldots\ldots \ldots\ldots \ldots\ldots + 99! + 100!$ By 12

3. Attempt any *two* of the following : 10

(*a*) For integers a, b, c if $a \mid b$ and $a \mid c$, then prove that $a \mid (bx + cy) \mid$ for arbitrary integers x and y

(*b*) Show that the number $\sqrt{2}$ is irrational

(*c*) Show that 41 divides $2^{20} - 1$

(*d*) Show that $2^{340} \equiv 1\ (mod\ 341)$.