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VA—56—2024

FACULTY OF SCIENCE AND TECHNOLOGY

B.Sc. (Third Year) (Sixth Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

(CBCS/New Pattern)

MATHEMATICS

Paper XVII-A

(Topology)

(Tuesday, 10-12-2024) **Time : 10.00 a.m. to 12.00 noon**

Time—Two Hours *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

(*iii*) All symbols carry their usual meanings.

(*iv*) Attempt either A or B for question 1 and 2.

1. (A) Attempt the following :

(*i*) Define discrete topology. If X is any set, show that the collection of all one-point subsets of X is a basis for the discrete topology on X. 10

(*ii*) Let X be a set, let β be a basis for a topology λ on X. Then prove that λ equals the collection of all unions of elements of β. 5

P.T.O.

Or

(*b*) Attempt the following :

(*i*) Define the product topology on X × Y. If β is a basis for the topology of X and **C** is a basis for the topology of Y, then show that the collection :

$$D = \{\beta \times c : \beta \in \beta \text{ and } c \in \mathbf{C}$$

is a basis for the topology of X × Y. 7

(*ii*) Let X be an ordered set in the order topology; let Y be a subset of X that is convex in X. Then prove that the order topology on Y is the same as the topology Y inherits as a subspace of X. 8

2. (*a*) Attempt the following :

(*i*) Let Y be a subspace of X. Then show that a set A is closed in Y iff it equals the intersection of a closed set of X with Y. 7

(*ii*) Define Hausdorff space. Hence show that the product of two Hausdorff spaces is a Hausdorff space. 8

Or

(*b*) Attempt the following :

(*i*) Let $f : A \to X \times Y$ be given by the equation $f(a) = (f_1(a), f_2(a))$. Then prove that f is continuous if and only if the functions $f_1 : A \to X$ and $f_2 : A \to Y$ are continuous. 8

(*ii*) If the sets C and D form a separation of X and if Y is a connected subspace of X, then show that Y lies entirely within either C or D. 7

3. Attempt any *two* of the following : 5 each

(*i*) Provide any *five* topologies on the set X = $\{a, b, c\}$ which are distinct.

(*ii*) Show that the collection :

$$S = \{\pi_1^{-1}(U) / U \text{ open in } X\} \cup \{\pi_2^{-1}(V) / V \text{ is open in } Y\}$$

is a subbasis for the product topology on X × Y.

(*iii*) Let X be a topological space, then prove that finite union of closed sets are closed.

(*iv*) Show that the interval (0, 1] is not compact.