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# VA—54—2024

## FACULTY OF ARTS AND SCIENCE

## B.Sc. (First Year) (First Semester) EXAMINATION

## NOVEMBER/DECEMBER, 2024

**(New Course)**

MATHEMATICS

Paper–I

(Calculus)

**(Tuesday, 10-12-2024)** **Time : 10.00 a.m. to 12.00 noon**

*Time*—2 *Hours* *Maximum Marks*—40

***N.B.*** :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. (*a*) State and prove the Leibnitz's theorem on $n$th derivative of the product of two functions. Also evaluate the $n$th derivative of the function. 15

$y = (ax + b)^m$

*Or*

(*b*) State and prove the Maclaurin's theorem on the expansion of functions. 8

(*c*) Find the angle of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4by$ at their point of intersection other than that the origin. 7

2. (*a*) State and prove the Cauchy's mean value theorem.

Hence if $f(x) = \sin x$, $F(x) = \cos x$, $x \in [\alpha, \beta]$, then show that,

$$\frac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \cot\theta,\ 0 < \alpha < \theta < \beta < \frac{\pi}{2}$$ 15

*Or*

(*b*) With the usual notations prove that, $f_{xy}(a, b) = f_{yx}(a, b)$ 8

(*c*) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then show that,

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x + y + z)^2}$$ 7

3. Attempt any *two* of the following : 10

(*a*) Prove that, $\frac{d^n}{dx^n}[e^{ax} \sin(bx + c)] = r^n.e^{ax} \sin(bx + c + n\phi)$ where $r = \sqrt{a^2 + b^2}$, $\phi = \tan^{-1}(b/a)$.

(*b*) Find the equations of the tangent and the normal at any point $(x, y)$ to the curve $\frac{x^m}{a^m} + \frac{y^m}{b^m} = 1$.

(*c*) State the generalized mean value theorem. Hence write the Taylor's remainder, remainder due to Cauchy and remainder due to Lagrange.

(*d*) If $u = \tan^{-1}\left\{\frac{(x^3 + y^3)}{(x - y)}\right\}$, $x \neq y$ show by Euler's theorem that,

$$x\frac{\partial u}{dx} + y\frac{\partial u}{\partial y} = \sin 2u.$$