This question paper contains **2** printed pages]

VA—41—2024

FACULTY OF SCIENCE

B.Sc. (Third Year) (Sixth Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

(CBCS/New Pattern)

MATHEMATICS

Paper–XVI

(Integral Transforms)

(Saturday, 7-12-2024) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. If $L[f(t)] = F(s)$, then prove that $L\left[\frac{1}{t} f(t)\right] = \int_8^{\infty} F(s)\, ds$ and hence find the Laplace transform of $f(t) = \int_0^t \frac{\sin t}{t} dt$.

Or

(*a*) Find the inverse Laplace transform of $\frac{s^2 + s + 2}{s^{3/2}}$ 8

(*b*) Find the inverse Laplace transform of $\frac{s}{s^2 + 4s + 13}$ 7

P.T.O.

2. (*a*) Using Laplace transforms, find the solution of the initial value problem :

$$y'' - 4y'' + 4y = 64 \sin 2t$$ 8

$$y(0) = 0, \; y'(0) = 1$$

(*b*) Solve the initial value problem. 7

$$2y'' + 5y' + 2y = e^{-2t}$$

$$y(0) = 1, \; y'(0) = 1$$

Using the Laplace transforms.

Or

Prove the Fourier Integral Theorem.

$$f(x) = \frac{1}{\pi}\int_0^{\infty}\int_{-\infty}^{\infty} f(t) \,.\, \cos u(t - x) \, dt du$$ 15

3. Attempt any *two* : 5 marks each

(*a*) Find the Laplace Transform of $t \sin at$.

(*b*) Find inverse Laplace transform of $\frac{s^2 + 3}{s\,(s^2 + 9)}$.

(*c*) Using the Laplace transforms, find the solution of the initial value problem :

$$y'' + 25y = 10 \cos 5t$$

$$y(0) = 2, \; y'(0) = 0$$

(*d*) If $F_1(s)$ and $F_2(s)$ are Fourier transforms of $f_1(x)$ and $f_2(x)$ respectively, then prove that

$$F\,[af_1(x) + bf_2(x)] = aF_1(s) + bF_2(s)$$