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VA—26—2024

FACULTY OF SCIENCE

B.Sc. (Third Year) (Sixth Semester) EXAMINATION

NOVEMBER/DECEMBER, 2024

(CBCS/New Pattern)

MATHEMATICS

Paper–XV

(Complex Analysis)

(Thursday, 5-12-2024) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Discuss the process to find the nth roots of complex number. Determine the nth roots of unity. 15

Or

(*a*) Suppose that, $f(z) = u(x, y) + iv(x, y)$ and that $f'(z)$ exists at a point $z_0 = x_0 + iy_0$. Then prove that the first order partial derivatives of u and v must exist at (x_0, y_0) and satisfy the Cauchy-Riemann equations. Also prove that $f'(z_0) = u_x + iv_x$; where these partial derivatives are to be evaluated at (x_0, y_0). 8

(*b*) (*i*) Find the principal value of $(-i)^i$. 7

(*ii*) Find the principal branch of $z^{2/3}$.

P.T.O.

2. Suppose that a function $f(z)$ is analytic throughout a disk $|z - z_0| < R_0$ centered at z_0 and with radius R_0. Then prove that $f(z)$ has the power series representation $f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n$; where $a_n = \frac{f^n(z_0)}{n!}, n = 0, 1, 2, \ldots\ldots.$ 15

Or

(*a*) Let C denote a contour of length L and suppose that a function $f(z)$ is piecewise continuous on C. If M is a non-negative constant such that $|f(z)| \leq M$ for all points z on C at which $f(z)$ is defined, then prove that :

$$\left|\int_C f(z)dz\right| \leq ML$$

Hence show that $\left|\int_C \frac{z+4}{z^3-1} dz\right| \leq \frac{6\pi}{7}$; where C is the arc of the circle $|z| = 2$ from 2 to $z = 2i$ that lies in the first quadrant. 8

(*b*) Evaluate the integral $I = \int_C \bar{z}\, dz$; where C is the right hand half of the circle $|z| = 2$ from $z = -2i$ to $z = 2i$. 7

3. Attempt any *two* of the following : 10

(*a*) Show that, a limit of a function $f(z)$ (if exists) at a point z_0 is unique

(*b*) Show that $u(x, y) = 2x - x^3 + 3xy^2$ is harmonic in some domain and find harmonic conjugate $v(x, y)$.

(*c*) Evaluate :

$$I = \int_0^{1+i} z^2 dz.$$

(*d*) Let z_0 be any point interior to a positively oriented simple closed contour C. For $f(z) = 1$, prove that :

(*i*) $$\int_C \frac{dz}{z - z_0} = 2\pi i$$

(*ii*) $$\int_C \frac{dz}{(z - z_0)^{n+1}} = 0.$$