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PA—94—2024

FACULTY OF SCIENCE

B.Sc. (Fourth Semester) EXAMINATION

APRIL/MAY, 2024

(New Pattern)

MATHEMATICS

Paper XI

(Partial Differential Equations)

(Monday, 29-04-2024) **Time : 2.00 p.m. to 4.00 p.m.**

Time—Two Hours *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. If $P_p + Q_q = R$ is a linear equation in p and q, where P, Q and R being functions of x, y, z, then discuss the method that the equation : 15

$$P\frac{\partial u}{\partial x} + Q\frac{\partial u}{\partial y} + R\frac{\partial u}{\partial z} = 0$$

is equivalent to the form :

$$P_p + Q_q = R$$

Solve :

$$(x^2 - yz)\, p + (y^2 - zx)\, q = z^2 - xy.$$

Or

(*a*) Discuss the method of finding complementary function of homogeneous equation : 8

$$(a_0 D^n + a_1 D^{n-1} D^1 + \ldots\ldots\ldots + a_n D^{1n})\, z = f(x, y).$$

(*b*) Solve : 7

$$p^2 + q^2 = 1.$$

P.T.O.

2. Explain Monge's method for solving the non-linear equation of second order : 15

$$Rr + Ss + Tt = V$$

where R, S, T and V are the functions of x, y, z, p, q and $r = \frac{\partial^2 f}{\partial x^2}$, $s = \frac{\partial^2 f}{\partial x \partial y}$ and $t = \frac{\partial^2 f}{\partial y^2}$.

Solve :

$$r - t + p - q = 0.$$

Or

(*a*) Find the solution of $\frac{\partial^2 u}{\partial x^2} = h^2 \frac{\partial^2 u}{\partial t}$ for which $u(0, t) = u(l, t) = 0$, $u(x, 0) = \sin \frac{\pi x}{l}$ by method of variable separable. 8

(*b*) Derive solution of wave equation :

$$\frac{\partial^2 y}{\partial t^2} = C^2 \frac{\partial^2 y}{\partial x^2}$$

by D'Alembert's method. 7

3. Attempt any *two* of the following : 10

(*a*) Form the partial differential equation by eliminating the arbitrary constants from :

$$z = (x + a)(y + b)$$

(*b*) Solve :

$$\frac{\partial^2 z}{\partial x^2} - 5 \frac{\partial^2 z}{\partial x \, \partial y} + 6 \frac{\partial^2 z}{\sigma y^2} = e^{x+y}.$$

(*c*) Obtain the solution of wave equation :

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

using the method of separable variable.

(*d*) Solve :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

which satisfies the conditions :

$$u(0,\ y) = u(l,\ y) = u(x,\ 0) = 0$$

and $u(x,\ a) = \sin \frac{n\pi x}{l}$.