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PA—81—2024

FACULTY OF SCIENCE

B.Sc. (Fourth Semester) EXAMINATION

APRIL/MAY, 2024

(New Pattern)

MATHEMATICS

Paper–X

(Ring Theory)

(Tuesday, 23-04-2024) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Define field, integral domain and show a ring R is without zero divisors if and only if the cancellation laws hold in R. 15

Or

(*a*) Show that an arbitrary intersection of left ideals of a ring is a left ideal of the ring. 8

(*b*) If F is a field, then $F[x_1, x_2, ... x_n]$ is an integral domain. 7

2. Let F be a field, $f(x)$ and $g(x)$ be any two polynomials in $F[x]$, not both of which are zero. Then $f(x)$ and $g(x)$ have a greatest common divisor $d(x)$ which can be expressed in the form 15

$$d(x) = m(x)\ f(x) + n(x)\ g(x), \text{ for polynomials } m(x) \text{ and } n(x) \text{ in } F(x).$$

P.T.O.

Or

(*a*) Show that ring of integers is a principal ideal ring. 8

(*b*) Show that the ring of integers is a Euclidean ring. 7

3. Attempt any *two* of the following : 10

(*a*) If an ideal U of a ring R contains a unit of R, then U = R.

(*b*) Find all units of the integral domain of Gaussian integers.

(*c*) If a, b, c, d are elements of a ring R, then evaluate $(a + b)(c + d)$.

(*d*) Show that the set of matrices $\begin{bmatrix} a & b \\ o & c \end{bmatrix}$ is a subring of the ring 2 × 2 matrices with integral elements.