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PA—74—2024

FACULTY OF SCIENCE

B.Sc. (Second Year) (Third Semester) EXAMINATION

APRIL/MAY, 2024

(New Course)

MATHEMATICS

Paper-VII

(Group Theory)

(Monday, 22-04-2024) **Time : 2.00 p.m. to 4.00 p.m.**

Time—Two Hours *Maximum Marks*—40

N.B. :— (*i*) Attempt *All* questions.

(*ii*) Illustrate your answers with suitably labelled diagrams wherever necessary.

(*iii*) Figures to the right indicate full marks.

1. Prove that : 15

(*i*) The inverse of each element of a group is unique.

(*ii*) If the inverse of a is a^{-1}, then the inverse of a^{-1} is a i.e. $(a^{-1})^{-1} = a$.

(*iii*) $(ab)^{-1} = b^{-1}a^{-1}, \ \forall \ a, b \in G$.

Or

(*a*) State and prove Lagrange's theorem. 8

(*b*) Prove that any two right (left) cosets of a subgroup are either disjoint or identical. 7

2. Prove that the set of all automorphisms of a group forms a group with respect to composite of functions as the composition. 15

P.T.O.

Or

(*a*) Show that "Congruence modulo *m*" is an equivalence relation on set of integers I. 8

(*b*) Prove that the order of an element of a group is the same as that of its inverse a^{-1}. 7

3. Attempt any *two* of the following : 5 each

(*a*) Show that the set I of all integrs ...., –4, –3, –2, –1, 0, 1, 2, 3, 4 .... is a group with respect to the operation of addition of integers.

(*b*) Show that if a, b are any two elements of a group G, then $(ab)^2 = a^2b^2$ if and only if G is abelian.

(*c*) If H is a subgroup of G and N is a normal subgroup of G, show that $H \cap N$ is a normal subgroup of H.

(*d*) Prove that every cyclic group is an abelian group.