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PA—68—2024

FACULTY OF SCIENCE & ARTS

B.A./B.Sc. (Second Year) (Fourth Semester) EXAMINATION

APRIL/MAY, 2024

(New Course)

MATHEMATICS

Paper-IX

(Real Analysis-II)

Saturday, 20-04-2024) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (1) Attempt *all* questions.

(2) Figures to the right indicate full marks.

1. Prove that a necessary and sufficient condition for the integrability of a bounded function f is that to every $\in > 0$, there corresponds $\delta > 0$ such that for every partition P of $[a, b]$ with norm $\mu(P) < \delta$: 15

$$U(P, f) - L(P, f) < \in$$

Or

(*a*) Prove that a function f is integrable over $[a, b]$ iff there is a number I lying between $L(P, f)$ & $U(P, f)$ such that for any $\in > 0$, $\exists$ a partition P of $[a, b]$ such that : 8

$$|U(P, f) - I| < \in \text{ and}$$

$$|I - L(P, f)| < \in$$

(*b*) Prove that every integrable continuous function is integrable. 7

P.T.O.

2. If f and g be two positive function such that $f(x) \leq g(x)$, for all x in $[a, b]$ then : 15

(*i*) $\int_a^b f dx$ converges if $\int_a^b g\,dx$ converges.

(*ii*) $\int_a^b g\,dx$ diverges, and if $\int_a^b f dx$ diverges and also test the convergenic of $\int_0^1 \frac{dx}{\sqrt{1-x^3}}$.

Or

(*a*) If f and g are positive in $[a, x]$ and $\lim_{x\to\infty} \frac{f}{g} = l$, where l is a non-zero finite number, then two integral $\int_a^\infty f\,dx$ and $\int_a^\infty g\,dx$ converge or diverge together. Also if $f/g \to 0$ and $\int_a^\infty g\,dx$ converges then prove that $\int_a^\infty f\,dx$ converges and if $f/g \to \infty$ and $\int_a^\infty g\,dx$ diverges, then $\int_a^\infty f\,dx$ diverges. 8

(*b*) If ϕ is bounded of monotonic in $[a, \infty]$ and $\int_a^\infty f\,dx$ is convergent at ∞, then prove that $\int_a^\infty f\phi\,dx$ is convergent at ∞. 7

3. Atempt any *two* :

(*a*) Show that x^2 is integrable on any interval $[0, k]$. 5

(*b*) Compute $\int_{-1}^{1} f\,dx$, where $f(x) = |x|$. 5

(*c*) Examine the convergence of $\int_{0}^{1} \frac{dx}{x^2}$. 5

(*d*) Show that $\int_{1}^{\infty} \frac{\sin x}{\rho}\,dx$ converges absolutely if P > 1. 5