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PA—63—2024

FACULTY OF ARTS/SCIENCE

B.Sc. (Third Year) (Fifth Semester) EXAMINATION

APRIL/MAY, 2024

(CBCS/New Pattern)

MATHEMATICS

Paper XIII

(Linear Algebra)

(Saturday, 20-4-2024) **Time : 10.00 a.m. to 12.00 noon**

Time—Two Hours *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. State and prove the Rank-Nullity Theorem. 15

Or

(*a*) Let U and W be two subspaces of a vector space V and Z = U + W then prove that $Z = U \oplus W$ iff for any vector $z \in Z$ can be expressed uniquely as the sum $z = u + w$, $u \in U$, $w \in W$. 8

(*b*) In a vector space V suppose $\{v_1, v_2, ..., v_n\}$ is an ordered set of vectors with $v_1 \neq 0$. The set is L.D. iff one of the vectors $v_2, v_3, ..., v_n$ say v_k belongs to the span of $v_1, v_2, ..., v_k$, i.e. $v_k \in [v_1, v_2, ..., v_{k-1}]$. 7

2. Let V be an inner product space then for arbitrary vectors u and v in V and scalars α, prove that : 15

(*i*) $\| \alpha u \| = \| \alpha \| \; \| u \|$

(*ii*) $\| u . v \| \leq \| u \| . \| v \|$

(*iii*) $\| u + v \| \leq \| u \| + \| v \|$.

P.T.O.

Or

(*a*) Every real vector space of dimension p is isomorphic to V_p. 8

(*b*) Let T : U → V and S : U → V be two linear transformations, then show that the mappings M : U → V defined by $M(u) = S(u) + T(u)$ and P : U → V defined by $P(u) = \alpha(S(u))\ \forall\ u \in U$ are linear. 7

3. Attempt any *two* of the following : 5 marks each

(*a*) If U and W be two subspaces of a vector space V, then prove that their intersection $U \cap W$ is also a subspace of V.

(*b*) Let T : U → V be a linear map, then prove that :

(*i*) $T(O_U) = O_V$

(*ii*) $T(-u) = -T(u)$.

(*c*) Let T : $P_2 \to V_3$ defined by :

$$T(\alpha_0 + \alpha_1 x + \alpha_2 x^2) = (\alpha_0, \alpha_1, \alpha_2).$$

Show that T is linear and non-singular.

(*d*) Let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

Find the eigenvalues of A.