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PA—61—2024

FACULTY OF SCIENCE/ARTS

B.Sc./B.A. (First Year) (Second Semester) EXAMINATION

APRIL/MAY, 2024

MATHEMATICS

Paper III

(Calculus–II)

(Saturday, 20-4-2024)

Time : 10.00 a.m. to 12.00 noon

Time—Two Hours

Maximum Marks—40

N.B. :— (i) Attempt All questions.

(ii) Figures to the right indicate full marks.

1. Prove that :

15

$$\int x^m (a + bx^n)^p dx = \frac{x^{m+1} (a + bx^n)^p}{np + m + 1} + \frac{apn}{np + m + 1} \int x^m (a + bx^n)^{p-1} dx.$$

Or

(a) Integrate :

8

$$\frac{Lx + M}{(Ax^2 + 2Bx + C)^n},$$

where n is a positive integer different from 1.

(b) Integrate :

7

$$\frac{dy}{(y^2 + 1)^2}.$$

P.T.O.

2. Prove that reduction formula :

15

$$\int \sin^m x \cos^n x \, dx = \frac{-\sin^{m-1} x \cdot \cos^{n+1} x}{m+n} + \frac{m-1}{m+n} \int \sin^{m-2} x \cdot \cos^n x \, dx.$$

Or

(a) Prove that :

8

$$\beta(m, n) = \frac{\overline{m} \cdot \overline{n}}{\overline{m+n}}.$$

(b) Evaluate :

7

$$\int_0^a \int_0^b (x^2 + y^2) \, dx \, dy.$$

3. Attempt any *two* of the following :

5 marks for each

(a) Evaluate :

$$\int \frac{1}{x^2 + 3x + 4} \, dx.$$

(b) Integrate :

$$\frac{x+1}{\sqrt{x^2 - x + 1}}.$$

(c) Prove that :

$$\int_0^{2a} f(x) \, dx = \begin{cases} 2 \int_0^a f(x) \, dx, & \text{if } f(2a-x) = f(x) \\ 0, & \text{if } f(2a-x) = -f(x) \end{cases}.$$

(d) Prove that :

$$\overline{1/2} = \sqrt{\pi}.$$