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PA—59—2024

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Second Year) (Third Semester) EXAMINATION

MARCH/APRIL, 2024

(New Pattern)

MATHEMATICS

Paper VI

(Real Analysis-I)

(Friday, 19-04-2024) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. (*a*) Prove that, a countable union of countable sets is countable. Also show that set of rational numbers in [0, 1] is countable. 15

Or

(*b*) Attempt the following :

(*i*) Prove that, every convergent sequence is bounded and has unique limit. 8

(*ii*) Show that there is no rational number whose square is 2. 7

P.T.O.

2. (*a*) Attempt the following :

(*i*) If $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ are three sequences such that : 8

(1) $a_n \le b_n \le c_n, \forall\ n$

(2) $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} c_n = l,$

then prove that $\lim\limits_{n\to\infty} b_n = l$

(*ii*) Show that the sequence $\{S_n\}$ where

$$S_n = 1 + \frac{1}{2!} + \frac{1}{3!} + \ldots + \frac{1}{n!}, n \leftarrow N$$

is convergent. 7

Or

(*b*) Attempt the following :

(*i*) If Σu_n and Σv_n are two positive term series and there exist a positive integer m such that 8

$$\frac{u_n}{u_{n+1}} \ge \frac{v_n}{v_{n+1}}, \forall\ n \ge m,$$

then prove that

(1) Σu_n convergent if Σv_n is convergent

(2) Σv_n is divergent if Σu_n is divergent

(*ii*) Show that the series :

$$\frac{1.2}{3^2.4^2} + \frac{3.4}{5^2.6^2} + \frac{5.6}{7^2.8^2} + ...$$

is convergent. 7

3. Attempt any *two* of the following : 10

(*a*) Write the properties for a set to be a complete-ordered field.

(*b*) If $\{a_n\}$, $\{b_n\}$ be two sequences such that $\lim\limits_{n \to \infty} a_n = a$, $\lim\limits_{n \to \infty} b_n = b$, then prove that :

$$\lim_{n \to \infty} (a_n + b_n) = \lim_{n \to \infty} (a_n) + \lim_{n \to \infty} (b_n) = a + b$$

(*c*) State the Raabe's test and the logarithmic test.

(*d*) Test for the convergence, the series whose *n*th term is $\left\{(n^3 + 1)^{1/3} - n\right\}$.