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PA—56—2024

FACULTY OF SCIENCE AND TECHNOLOGY

B.Sc. (Third Year) (Sixth Semester) EXAMINATION

APRIL/MAY, 2024

(CBCS/New Pattern)

MATHEMATICS

Paper–XVII

(Topology)

(Friday, 19-04-2024) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) Attempt either (A) or (B) for Q. No. **1** and **2**.

(*ii*) *All* symbols carry usual meanings.

(*iii*) Figures to the right indicate full marks.

1. (A) Attempt the following :

(*a*) Define an equivalence relation and hence prove that two equivalence classes E and E' are either disjoint or equal. 8

(*b*) Define finer and coarser topologies and hence show that if β and β' be a bases for the topologies τ and τ' respectively, on X, then the following are equivalent : 7

(*i*) τ' is finer than τ.

(*ii*) For each $x \in X$ and each basis element $B \in \beta$ containing x, there is a basis element $B' \in \beta'$ such that $x \in B' \subset B$.

P.T.O.

Or

(B) Attempt the following

(*a*) Let X be an ordered set in the order topology. Let Y be a subset of X that is convex in X. Then prove that the order topology on Y is same as the topology Y inherits as a subspace of X. 8

(*b*) Define open maps and show that $\pi_1 : X \times Y \to X$ and $\pi_2 : X \times Y \to Y$ are open maps. 7

2. (A) Let X be a topological space. Then prove that the following conditions holds :

(*i*) ϕ and X are closed. 3

(*ii*) Arbitrary intersection of closed sets are closed. 6

(*iii*) Finite union of closed sets are closed. 6

Or

(B) Attempt the following :

(*a*) Show that subspace of Hausdorff space is Hausdorff. 8

(*b*) Let $f : A \to X \times Y$ be given by the equation $f(a) = (f_1 (a), f_2(a)$. Then prove that f is continuous if and only if the function $f_1 : A \to X$ and $f_2 : A \to Y$ are continuous. 7

3. Attempt any *two* of the following : 5 each

(*a*) Let X be a set; let τ_f be the collection of all subsets U of X such that X–U either is finite or is all of X. Then show that τ_f is a topology on X.

(*b*) If β is a basis for the topology on X and **C** is a basis for the topology on Y, then show that the collection :

$$D = \{B \times C / B \in \beta \text{ and } C \in \mathbf{C}\}$$

is a basis for the topology on X × Y.

(*c*) Define homeomorphism. If $f : R \to R$ be a function given by $f(x) = 3x + 1$, show that it is homeomorphism.

(*d*) Prove that the union of a collection of connected subspaces of X that have a point in common is connected.