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PA—50—2024

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Third Year) (Fifth Semester) EXAMINATION

APRIL/MAY, 2024

(CBCS/New Pattern)

MATHEMATICS

Paper–XII

(Metric Spaces)

(Thursday, 18-04-2024) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) Attempt *all* questions

(*ii*) Figures to the right indicate full marks.

1. Prove that every compact subset F, of a metric space (X, d), is closed. 15

Or

(*a*) Let (X, d) be any metric space. Prove that a subset F, of X, is closed if and only if it's complement in X is open. 8

P.T.O.

(*b*) Show that the function $d : R \times R \to R$ defined by $d(x, y) = |x - y|$; for all $x, y \in R$ is a metric on the set R of all real numbers. 7

2. Let Y be a subset of a metric space (X, d), then prove that the following are equivalent : 15

(*i*) Y is connected

(*ii*) Y cannot be expressed as disjoint union of two non-empty closed sets in Y.

Or

(*a*) Let (X, d) be a complete metric space and Y be a subspace of X, then prove that Y is complete if any only if it is closed in (X, d). 8

(*b*) Let (X, d_1) and (Y, d_2) be two metric spaces. Show that $f : X \to Y$ is continuous if and only if $F\left(\overline{A}\right) \subseteq \overline{F(A)}$, for every $A \subseteq X$. 7

3. Attempt any *two* of the following : 5 each

(*a*) Let (X, d) be any metric space. Show that the function d_1 defined by

$$d_1(x, y) = \frac{d(x, y)}{1 + d(x, y)}.$$

For all $x, y \in X$ is a metric on X.

(*b*) Prove that every compact subset A, of a metric space (X, d), is bounded.

(*c*) Prove that every convergent sequence is a Cauchy sequence.

(*d*) Discuss the connectedness of the subset :

$$D = \left\{(x, y) \mid x \neq 0,\ y = \sin\left(\frac{1}{x}\right)\right\}$$

of the Euclidean space R^2.