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**PA—41—2024**

**FACULTY OF SCIENCE**

**B.Sc. (Third Year) (Sixth Semester) EXAMINATION**

**APRIL/MAY, 2024**

**(CBCS/New Pattern)**

**MATHEMATICS**

**Paper—XVI**

**(Integral Transforms)**

**(Tuesday, 16-04-2024)**

**Time : 10.00 a.m. to 12.00 noon**

*Time—2 Hours*

*Maximum Marks—40*

*N.B. :—* (i) *All questions are compulsory.*

(ii) *Figures to the right indicate full marks.*

1. Let  $f(t)$  be a periodic function with period  $T$ , then :

15

$$L [f(t)] = \frac{\int_0^T e^{-st} f(t) dt}{1 - e^{-sT}}$$

and hence find the Laplace transform of the periodic function (saw tooth wave)

$$f(t) = \frac{kt}{T} \text{ for } 0 < t < T, \quad f(t + T) = f(t)$$

P.T.O.

Or

(a) Find the inverse Laplace transform of  $\frac{S}{S^2 + 4S + 13}$  8

(b) Find the inverse Laplace transform of 7

$$\frac{se^{-s/2} + \pi e^{-s}}{s^2 + \pi^2}$$

in terms of unit step functions.

2. (a) Using Laplace transforms, find the solution of the initial value problem

$$y'' - 4y' + 4y = 64 \sin 2t \quad 8$$

$$y(0) = 0, y'(0) = 1.$$

(b) Solve  $\frac{dx}{dt} + y = 0$  and  $\frac{dy}{dt} - x = 0$  under the condition  $x(0) = 1, y(0) = 0.$  7

Or

(a) State and prove Fourier integral theorem. 8

(b) Express the function :

$$f(x) = \begin{cases} 1 & \text{when } |x| \leq 1 \\ 0 & \text{when } |x| > 1 \end{cases}$$

as a Fourier integral. Hence evaluate  $\int_0^{\infty} \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda.$  7

P.T.O.

3. Attempt any *two* of the following :

5 each

(a) Find the Laplace transform of  $f(t)$  defined as :

$$f(t) = \frac{t}{k} \text{ when } 0 < t < k$$

$$= 1 \text{ when } t > k$$

(b) Obtain the inverse Laplace transform of  $\log \frac{s^2 - 1}{s^2}$ .

(c) Applying convolution, solve the following initial value problem

$$y'' + y = \sin 3t$$

$$y(0) = 0, y'(0) = 0$$

(d) If  $F(s)$  is the complex Fourier transform of  $f(x)$ , then

$$F \{f(x) \cos ax\} = \frac{1}{2} [F(s + a) + F(s - a)]$$