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PA—41—2024

FACULTY OF SCIENCE

B.Sc. (Third Year) (Sixth Semester) EXAMINATION

APRIL/MAY, 2024

(CBCS/New Pattern)

MATHEMATICS

Paper–XVI

(Integral Transforms)

(Tuesday, 16-04-2024) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Let $f(t)$ be a periodic function with period T, then : 15

$$L\ [f(t)] = \frac{\int_0^T e^{-st} f(t)dt}{1 - e^{-sT}}$$

and hence find the Laplace transform of the periodic function (saw tooth wave)

$$f(t) = \frac{kt}{T} \text{ for } 0 < t < T, \quad f\ (t + T) = f(t)$$

P.T.O.

Or

(*a*) Find the inverse Laplace transform of $\frac{S}{S^2 + 4S + 13}$ 8

(*b*) Find the inverse Laplace transform of 7

$$\frac{se^{-s/2} + \pi e^{-s}}{s^2 + \pi^2}$$

in terms of unit step functions.

2. (*a*) Using Laplace transforms, find the solution of the initial value problem

$$y'' - 4y' + 4y = 64 \sin 2t$$ 8

$$y(0) = 0,\ y'(0) = 1.$$

(*b*) Solve $\frac{dx}{dt} + y = 0$ and $\frac{dy}{dt} - x = 0$ under the condition $x(0) = 1,\ y(0) = 0.$ 7

Or

(*a*) State and prove Fourier integral theorem. 8

(*b*) Express the function :

$$f(x) = \begin{cases} 1 & \text{when } |x| \leq 1 \\ 0 & \text{when } |x| > 1 \end{cases}$$

as a Fourier integral. Hence evaluate $\int_0^\infty \frac{\sin \lambda \cos \lambda x}{\lambda}\, d\lambda$. 7

P.T.O.

3. Attempt any *two* of the following : 5 each

(*a*) Find the Laplace transform of $f(t)$ defined as :

$$f(t) = \frac{t}{k} \text{ when } 0 < t < k$$

$$= 1 \text{ when } t > k$$

(*b*) Obtain the inverse Laplace transform of $\log \frac{s^2 - 1}{s^2}$.

(*c*) Applying convolution, solve the following initial value problem

$$y'' + y = \sin 3t$$

$$y(0) = 0,\ y'(0) = 0$$

(*d*) If F(s) is the complex Fourier tranform of $f(x)$, then

$$\text{F}\ \{f(x)\cos ax\} = \frac{1}{2}[\text{F}(s + a) + \text{F}(s - a)]$$