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PA—26—2024

FACULTY OF SCIENCE

B.Sc. (Third Year) (Sixth Semester) EXAMINATION

APRIL/MAY, 2024

(CBSE/New Course)

MATHEMATICS

Paper–XV

(Complex Analysis)

(Saturday, 13-04-2024) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Suppose that, $z = x + iy$ and $f(z) = u(x, y) + iv\ (x, y)$, 15

$z_0 = x_0 + iy_0,\ w_0 = u_0 + iv_0$

Then prove $\lim\limits_{z \to z_0} f(z) = w_0$

if and only if

$$\lim_{(x,\, y) \to (x_o, y_0)} u(x, y) = u_0 \text{ and}$$

$$\lim_{(x,\, y) \to (x_o, y_0)} v(x, y) = v_0$$

P.T.O.

Also show $\lim_{z \to 0} f(z)$ does not exist, for $f(z) = \dfrac{z}{\boldsymbol{z}}$.

Or

(*a*) Define entire function. Suppose that a function $f(z) = u(x, y) + iv(x, y)$ and its conjugate $\overline{f(z)} = u(x, y) - iv(x, y)$ are both analytic in a given domain D. Then show that $f(z)$ must be constant throughout D. 8

(*b*) (*i*) Show that :

$\log(i^3) \neq 3 \log(i)$. 7

(*ii*) If $z = -1 - \sqrt{3}\,i$, then find the value of $\log\left(-1 - \sqrt{3}i\right)$.

2. State and prove the fundamental theorem of algebra. 15

Or

(*a*) Show that, $\int_{-C} f(z)\,dz = -\int_{C} f(z)\,dz$.

Find the value of the integral $I = \int_{C} \bar{z}\,dz$, where C is the right hand half of the circle $|z| = 2$. 8

(*b*) Let C denote a contour of length L, and suppose that a function $f(z)$ is piecewise continuous on C. If M is a non-negative constant such that

$|f(z)| \leq$ M for all points z on C at which $f(z)$ is defined then prove that : 7

$$\left| \int_C f(z)\, dz \right| \leq ML.$$

3. Solve any *two* of the following :

(*a*) Find the cube root of 1. 5

(*b*) Solve the following :

$$\exp\left(\frac{2+\pi i}{4}\right) = \sqrt{\frac{e}{2}}.$$ 5

(*c*) Evaluate the following integral : 5

$$\int_0^{\pi/6} e^{i2t}\, dt$$

(*d*) Let C be the positively oriented circle $|z| = 2$. Then evaluate the integral

$$\int_C \frac{z\,dz}{(g - z^2)(z + i)}.$$ 5