INDIAN STREAMS RESEARCH JOURNAL

ISSN: 2230-7850
IMPACT FACTOR : 4.1625(UIF)
VOLUME - 6 | ISSUE - 12 | JANUARY – 2017

EXISTENCE OF LOCALLY ATTRACTIVE SOLUTION TO NONLINEAR QUADRATICVOLTERRA INTEGRAL EQUATION

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$$\int_0^l K(x_1,x)u(x)\,dx = v(x_1)$$

$$\frac{1}{k}\int_0^l K(x_1,x)u(x)\,dx = u(x_1)$$

$$u(x_1) - \int_0^l K(x_1,x)u(x)\,dx = f(x)$$

ABSTRACT: -

Fractional calculus developed only as the theoretical field of mathematics. Fractional differential equations play an important role in the study of various physical chemical and biological phenomenon's many researchers are attracted from the field of theory methods and application of fractional differential equation. The research have developed arevarious method to obtain of techniques to obtained approximate solutions of both linear and nonlinear fractional differential and integral equations. In recent year we see that monograph's Kalibas, Lakashminath [4], Podlibuny and Abbas [6-8], banas [10, 11] Darwish [12-13], Dhage [20-24] and B.D.Karande [1] and there references. In this paper we study the existence of locally attractive solution is of the following nonlinear quadratic volterra integral equation of fractional order.

$$x(t) = [f(x(t))][q(t) + \frac{1}{\Gamma(\alpha)}\int_0^t \frac{g\big(t,x(s)\big)}{(t-s)^{1-\alpha}}\,ds] \qquad 1.1$$

$for\ all\ t\epsilon R_+\ and\ \alpha(0,1)$In the space of real function defined continuous bounded or unbounded intervalsR_+. In the next section we give some basic definition and theorem which are used in further in this paper.We proceed the generalization the results are obtained.

PRELIMINARIES

Let $L^1(a,b)$bethe lebesuge intergable function. On interval (a, b) then let $x \in L^1(a,b) and\ \alpha > 0$ be a fixed number of Riemann-Liouville fractional integral order $\alpha\ of\ function\ x(t) then$

$$I^{\alpha}x(t) = \frac{1}{\Gamma(\alpha)}\int_0^t \frac{x(s)}{(t-s)^{1-\alpha}}\delta s \quad , t\epsilon(a,b) \qquad 2.1$$

Where $\Gamma(\alpha)$-does gamma function [Kalibus]. It may show that fractional integral space $L^1(a,b)$ into it self has some of the properties [see 10-12].

Let $X = BC(R_{+)}$ be the space continuous bounded and let Ω be the subset of X. Let a mappingA:X→X on the operator consider the following equations namely

$$x(t) = (px)(t) \qquad 2.2$$

$for all\ t\epsilon R_+$.Below given different characterization of solution for the operator on R_+. We need the following definitions in the sequel.

2.1Definition:We say that the equation 2.2 are locally attractive if there exists an $x \in BC(R_+)and\ r > 0$ such that for all solution$x = x(t) and\ y = (t)$ of equation 2.2belong $B_r\ (x_0 r)\Omega \quad for\ t\ \geq T$.

$$\lim_{t\to\infty} \ (x(t) - y(t)) = 0 \qquad 2.3$$

2.2Defination: an operator P: X→X iscalled Lipschitz if there exist constant k such that $\|px - py\| \leq \|X - Y\|\ for\ all\ x, y\ \in X$ the constant is called Lipschitz constant of P on X.

2.3Defination: [Dugundji and Granas] an operator Banach space X into itself is called compact subset of S. If any bounded set of X P(S) is relatively compact subset of X. If P is continuous and compact then it is called completely continuous on X.

We seek the solution of (1.1) in the space $BC(R_+)$ is continuous and bounded real valued function defined on R_+ .Define a standard supremum norm$\|.\|$ and multiplication "." In $BC(R_+)$ by

$$\|x\| = Sup\ \{\ x(t): t \in R_+\},$$

$$(x, y)(t) = x(t)y(t)\ t \in R_+ \qquad 2.4$$

Clearly$BC(R_{+)}$ become Banach space with Banach space with respect to above norm and then multiplication in it by $L^1(R_+)$ we denote the space of Lebesguge integrable function on R_+ with the norm $\|.\|_{L^1}$ defined by

$$\|X\|_{L^1} = \int_0^{\infty}|x(t)|\ dt$$

We employ a hybrid fixed point theorem of Dhage [14] for proving the existing results,

2.4Theorem [Dhage14] :Let s be closed convex and bounded subset of Banach space X and let $F: G: S \to S$ be two operators satisfying

a) F is Lipschitz with Lipschitz constant K

b) G is completely continuous.

c) $FxGx \in S\ for\ all\ x \in S$.

d) $M_k < 1\, where\, M = \|G(S)\| = Sup\,\{\|G(x)\|: x \in S\}$
Then the operator equation,

$$FxGx = x$$

has a solution . Aset of all solution in compacts. In case the lim. (2.3) is uniform with respect to the set $B(x_0 r)\Omega$ i.e. when each $\in > 0$ there exist T>0.such that$|x(t) - y(t)| < \in\ \forall x, y\ \in B(x_0 r) \cap \Omega\ and$ $t \geq T$We say that the solution is uniformly locally attractive.

2.5Defination: The solution $X = X(t)$ in equation 2.3 is said to the globally attractive if equation 2.4 holds for each solution y(t) of equation 2.3.

Existence Result

We consider the following hypothesis in the sequel.

$\boldsymbol{H_1}$ The function $f: R_+ \to R$ is continuous and there exists a bounded function$l: R_+ \to R$ with bound L satisfying

$$|f(t,x) - f(t,y)| \leq l(t)|x - y| for\ all\ t \in R_+ and\ x, y\ \in R.$$

$\boldsymbol{H_2}$ The function $f_1: R_+ \to R$ defined $f_1 = |f(t,0)|$ is bounded with $f_0 = Sup\{f_1(t): R_+\}$.

$\boldsymbol{H_3}$ The function $q: R_+$ is continuous and $\lim_{t\to\infty} q(t) = 0$.

$\boldsymbol{H_4}$ The function $g: R_+ \to R$ is continuous moreover there exist a function m:$R_+ \to R_+$ belong continuous on $R_+$and function $h: R_+ \to R_+$ with h (0) = 0 such that

$$|g(t,s,x) - g(t,s,y)| \leq m(t)h(|x - y|)$$

$$for\ all\ t, s \in R\ such\ that\ s \leq t\ and\ for\ all\ x, y\ \in R.$$

Further suppose let's define the function$g_1(t) = \max\{|g(t,s,o)| : 0 \leq s \leq t\}$ obviously the function $g_1$is continuous R_+.

$\boldsymbol{H_5}$ The function a, b:$R_+ \to R_+$ then defined formula$a(t) = m(t)t^\alpha$, $b(t) = g_1(t)t^\alpha$ are bounded on R_+ and vanish at infinity that is ,
t$\lim_{t\to\infty} a(t) = \lim_{t\to\infty} b(t) = 0$.

3.1Remark Note that the hypothesis $(H_3\,)\ and\ (H_5)$ holds than there exist constant $K_1 > 0\ and\ K_2 > 0$

$$K_1 = Sup\{q(t) : t \in R_+\} \qquad 3.1$$

$$K_2 = Sup\left\{\frac{a(t)h(r)+b(t)}{\Gamma(r+1)} : t, r\epsilon R_+\right\} \qquad 3.2$$

3.2 Theorem: Assume that the hypothesis $H_1 - H_5$ holds furthermore if$L(K_1 + K_2) < 1$, wehere $K_1 and\ K_2$are defined remark3.1 then 1.1 has at least one solution in the space $BC(R_+)$ moreover solution of (1.1) are locally attractive on R_+.

Set $X = BC(R_{+,}R)$ consider the closed at origin O and of the radius r where r =$\frac{f_0(K_1+K_2)}{1-L(K_1+K_2)} > 0$

Let's define the operators $F\alpha\ G\ on\ B_r(0)$ by,

$$Fx(t) = f\big(t(x)\big)$$

$$G(t) = q(t) + \frac{1}{\Gamma(\alpha)}\int_0^t \frac{g(t,s,x(s))}{(t-s)^{1-\alpha}}\,ds \qquad 3.3$$

$$for\ all\ t\epsilon R_+$$

Since the hypothesis (H_1) are holds the operator F is well defined the function FX is continuous and bounded view of hypothesis (H_4)therefore$F\alpha G$ define the operator $F.G: B_r(0) \to X$ will show that $F\alpha G$ satisfy the requirement of 2.4 on $B_r(0)$. Let $x, y \in B_r(o)$ be arbitrary then by hypothesis H_1 we get

$$\left|f\big(x(t)\big) - F\big(y(t)\big)\right| = \|f(t, x(t) - f(t, y(t)\|$$

$$\leq |(t)|(x)t - (y)t|$$

$$\leq L\|X - Y\|$$

$$for\ all\ t\ \in R_+\ taking\ superimum\ over\ t.$$

$$\|F(x) - F(y)\| \leq L\|X - Y\|$$

for all $x, y\ \in B_r(0)$.

This shows that F is Lipschitizion $B_r(0)$ with Lipschitz constant L.

II Now we show that G is continuous and compact operator $B_r(0)$. First we show that G is continuous on $B_r(0)$. Let's fix arbitrary $\in > 0$ and take $x, y\ B_r(0) such\ that\ \ \|x - y\| \leq \in$ then given

$$|G(x)(t) - G(y)(t)| \leq \frac{1}{\Gamma(\alpha)} \int_0^t \frac{\left|g(t, s(x(s))) - g, (t, s\big(y(s)\big))\right|}{(t - s)^{1-\alpha}} ds$$

$$\leq \frac{1}{\Gamma(\alpha)} \int_0^t \frac{m(t)h(|x(s) - y(s)|)}{(t - s)^{1-\alpha}} ds$$

$$\leq \frac{m(t)t^{\alpha}}{F(\alpha + 1)} h(r)$$

$$\leq \frac{a(t)}{F(\propto +1)} h(r) \qquad 3.4$$

Since h(r) is continuous on R_+ then its bounded on $R_+$and there exists a nonnegative constant $h^{\circledast}$ such that

$h^{\circledast} = \sup\{h(r): r > 0\}\, Hence$ In hypothesis (H_5) there exists T >0 such that $a(t) \leq \frac{\Gamma(\alpha+1)\epsilon}{h^{\circledast}}$ for t > T thus for t >T we derive

$|(G(x)(t) - G(y)(t))| < \in$ 3.5

Furthermore let's assume that $t \in [0, T]$ then evaluating similarly we obtain.

$$|(Gx)t - (Gy)t| \leq \frac{1}{\Gamma\alpha} \int_0^t \frac{|g(t, sx(s) - g(t, sy(s)|}{(t - s)^{1-\alpha}} ds$$

$$\leq \frac{T^{\alpha}}{\Gamma(\alpha + 1)} W_r^T(g\ \epsilon) \qquad 3.6$$

Where $W_r^T(g,\epsilon) = \sup\{\,|g(t,sx) - g(t,sy)| : t,s\,\epsilon[0,T],$

$$x,y\epsilon[-r\ r], |x-y| \leq \in$$

∴The uniform continuity of the function $g(t,s(x))$ on the set $[0,T]\times[0,T]\times[-r,r]$ we derive that $W_r^T(g,\in) \to 0\ as\ \epsilon \to 0$ Hence above establish factor we conclude that the operator G ball $B_r(0)$continuously into itself.

Now we show that G is compact $B_r(0)$. It is enough to show every sequence$\{Gx_n\}$ in $G(B_r(0))$has Cauchy subsequence. In view of hypothesis $H_3 and H_4$ we infer that,

$$|Gx_n(t)| \leq |q(t)| + \frac{1}{\Gamma(\alpha)}\int_0^t \frac{|g(t,s,x_n(s)|}{(t-s)^{1-\alpha}}ds$$

$$\leq |q(t)| + \frac{1}{\Gamma(\alpha)}\int_0^t \frac{|g(t,s,x_n(s) - g(t,s,0)|}{(t-s)^{1-\alpha}}ds + \frac{1}{\Gamma(\alpha)}\int_0^t \frac{|g(t,s,0)|}{(t-s)^{1-\alpha}}ds$$

$$\leq |q(t)| + \frac{1}{\Gamma(\alpha)}\int_0^t \frac{m(t)h(|x_n(s)|}{(t-s)^{1-\alpha}}ds + \frac{1}{\Gamma(\alpha)}\int_0^t \frac{g_1(t)}{(t-s)^{1-\alpha}}ds \quad 3.7 \leq |q(t)| + \frac{m(t)t^{\propto}}{\Gamma(\alpha+1)}h(r) + \frac{g_1(t)t^{\alpha}}{\Gamma(\alpha+1)}$$

$$\leq |q(t)| + \frac{a(t)h(r) + b(r)}{\Gamma(\alpha+1)}$$

$$\leq K_1 + K_2$$

for all $t \in R_+$ taking the superimum over t. We obtain $n \in N$.This shows that$\{G(x_n\}$ is uniformly bounded sequence in $G(B_r(0))$.

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