II semister
Paper - III
Integral Calculus

The basic concepts of *differential calculus* were covered in the preceding semister (paper) This paper will be devoted to *integral calculus*, which is the other broad area of calculus.

Anti-Derivatives

An anti-derivative of a function $f(x)$ is a new fun ction $F(x)$ such that

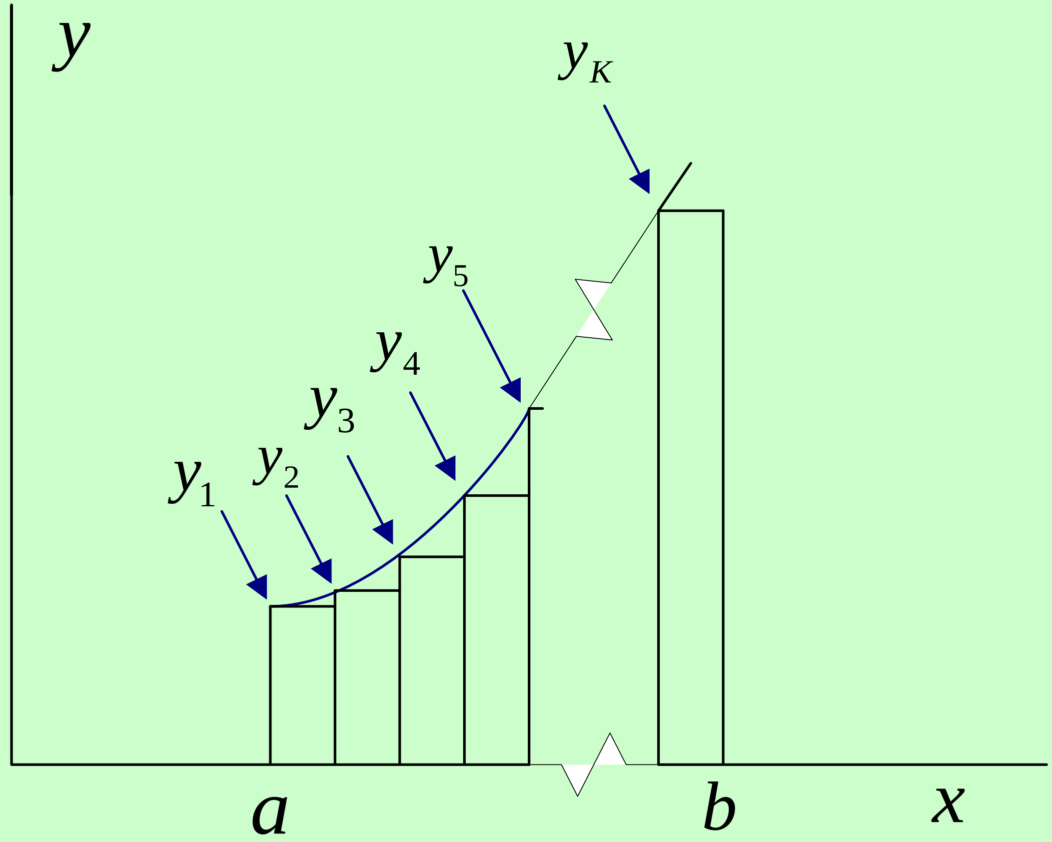
$$\frac{dF(x)}{dx} = f(x)$$

Indefinite and Definite Integrals

Indefinite $$\int f(x)dx$$

Definite $$\int_{x_1}^{x_2} f(x)dx$$

Definite Integral as Area Under the Curve



[illegible]

Exact Area as Definite Integral

$$\int_a^b f(x)\,dx = \lim_{\Delta x \to 0} \sum_k f(x_k)\,\Delta x$$

Definite Integral with Variable Upper Limit

$$\int_a^x y\,dx$$

More "proper" form with "dummy" variable

$$\int_a^x y(u)\,du$$

Tabulation of Integrals

$$F(x) = \int f(x)\,dx$$

$$I = \int_a^b f(x)\,dx$$

$$I = F(x)\Big|_a^b = F(b) - F(a)$$

Common Integrals.

$f(x)$	$F(x)=\int f(x)dx$	Integral Number
$af(x)$	$aF(x)$	I-1
$u(x)+v(x)$	$\int u(x)dx+\int v(x)dx$	I-2
a	ax	I-3
$x^n \qquad (n\neq -1)$	$\frac{x^{n+1}}{n+1}$	I-4
e^{ax}	$\frac{e^{ax}}{a}$	I-5
$\frac{1}{x}$	$\ln x$	I-6
$\sin ax$	$-\frac{1}{a}\cos ax$	I-7
$\cos ax$	$\frac{1}{a}\sin ax$	I-8
$\sin^2 ax$	$\frac{1}{2}x-\frac{1}{4a}\sin 2ax$	I-9

$\cos^2 ax$	$\frac{1}{2}x + \frac{1}{4a}\sin 2ax$	I-10
$x \sin ax$	$\frac{1}{a^2}\sin ax - \frac{x}{a}\cos ax$	I-11
$x \cos ax$	$\frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax$	I-12
$\sin ax \cos ax$	$\frac{1}{2a}\sin^2 ax$	I-13
$\sin ax \cos bx$ for $a^2 \neq b^2$	$-\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)}$	I-14
xe^{ax}	$\frac{e^{ax}}{a^2}(ax-1)$	I-15
$\ln x$	$x(\ln x - 1)$	I-16
$\frac{1}{ax^2+b}$	$\frac{1}{\sqrt{ab}}\tan^{-1}\left(x\sqrt{\frac{a}{b}}\right)$	I-17

Example

$$y = 12e^{4x}$$

$$z = \int 12e^{4x}\,dx = 12\frac{e^{4x}}{4} + C$$

$$= 3e^{4x} + C$$

Example

$$y = 12x\sin 2x$$

$$z = \int 12x\sin 2x\,dx$$

$$= 12\left(\frac{1}{(2)^2}\sin 2x - \frac{x}{2}\cos 2x\right) + C$$

$$= 3\sin 2x - 6x\cos 2x + C$$

Example

$$y = 6x^2 + \frac{3}{x}$$

$$z = \int \left(6x^2 + \frac{3}{x} \right) dx$$

$$= \int 6x^2 dx + \int \frac{3}{x} dx$$

$$= \frac{6x^3}{3} + 3\ln x + C$$

$$= 2x^3 + 3\ln x + C$$

In above examples that follow, deter mine the definite integral in each cas e as defined below.

$$I = \int_a^b y\,dx$$

Example

$$I = \int_0^{\pi} \sin x \, dx$$

$$I = \int_0^{\pi} \sin x \, dx = -\cos x \Big]_0^{\pi}$$

$$= -\cos \pi - (-\cos 0)$$

$$= -(-1) - (-1) = 2$$

Example

$$I = \int_0^1 8xe^{-2x}\,dx$$

$$I = \int_0^1 8xe^{-2x}\,dx$$

$$= 8\frac{e^{-2x}}{(2)^2}\left[-2x-1\right]_0^1$$

$$= 2e^{-2}\left[-2(1)-1\right] - 2e^{-0}\left[0-1\right]$$

$$= -6e^{-2} + 2 = 1.188$$

Displacement, Velocity, and Acceleration

$a(t) = \text{acceleration in meters/second}^2\ (\text{m/s}^2)$

$v(t) = \text{velocity in meters/second}\ (\text{m/s})$

$y(t) = \text{displacement in meters}\ (\text{m})$

$$\frac{dv}{dt} = a(t) \qquad dv = \left(\frac{dv}{dt}\right) dt = a(t)\,dt$$

$$\int dv = \int a(t)\,dt \qquad \int dv = v$$

Displacement, Velocity, and Acceleration

$$v = \int a(t)\,dt + C_1$$

$$\frac{dy}{dt} = v(t) \qquad dy = \left(\frac{dy}{dt}\right) dt = v(t)\,dt$$

$$y = \int v(t)\,dt + C_2$$

Example

$$y(t)=\int\left(10-10e^{-2t}\right)dt$$

$$=10t-\frac{10}{-2}e^{-2t}+C_2=10t+5e^{-2t}+C_2$$

$$y(0)=0=10(0)-\frac{10}{-2}e^{0}+C_2$$

$$C_2=-5$$

$$y(t)=10t+5e^{-2t}-5$$