UNIT II
SCHRODINGER'S EQUATION

B.Sc. T.Y.(Physics)

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SCHRODINGER'S EQUATION:
(TIME –DEPENDENT FORM)
BASIC PHYSICAL PRINCIPLE THAT CANNOT BE DERIVED FROM ANYTHING ELSE

- The wave function ψ of particle moving freely in + direction of X axis is specified by
- $\psi = A\, e^{-i\omega\left(t-\frac{x}{v}\right)}$ $\quad ----(1)$ But
- $\omega = 2\pi\upsilon$ and v= $\upsilon\lambda$
- $\psi = A\, e^{-i2\pi\upsilon\left(t-\frac{x}{\upsilon\lambda}\right)}$
- $\psi = A\, e^{-i2\pi\left(\upsilon t-\frac{x}{\lambda}\right)}$ ----------*(2)*

SCHRODINGER'S EQUATION:
(TIME –DEPENDENT FORM)

- The total energy E and momentum P of particle is
- $E = h\upsilon = 2\pi\hbar$ and $\lambda = \frac{h}{P} = \frac{2\pi\hbar}{P}$
- Equation (2) becomes (equation of free particle)
- $\psi = A\, e^{-\left(i/\hbar\right)(Et-Px)}$ -------(3)
- Eq. (3) Wave equivalent of unrestricted particle of The total energy E and momentum P of particle along + X axis

SCHRODINGER'S EQUATION:
(TIME –DEPENDENT FORM)

- For motion of particle under various restrictions.
- Differential equation (3) twice with *x* gives
- $$\frac{\partial^2\psi}{\partial x^2} = \frac{-P^2}{\hbar^2}\psi \quad ---(4)$$
- $\therefore\ P^2\ \psi = -\hbar^2 \frac{\partial^2\psi}{\partial x^2}$ ---(5)

SCHRODINGER'S EQUATION:
(TIME –DEPENDENT FORM)

- Differential equation (3) once with *t* gives
- $\frac{\partial\psi}{\partial t} = \frac{-E\psi}{\hbar}$
- $E\psi = \frac{-\hbar}{i}\frac{\partial\psi}{\partial t}$ ---(6)
- At a speed less than velocity of light, total energy E of particle is

SCHRODINGER'S EQUATION:
(TIME –DEPENDENT FORM)

- Total energy E = K.E. +P.E.
- $E = \frac{p^2}{2m} + \mathrm{U(x, t)}$ (7)
- U function represent influence of rest of universe
- Multiplying eq.n (7) by ψ on both sides
- $E\psi = \frac{p^2}{2m} + \mathrm{U}\psi$---(8)

SCHRODINGER'S EQUATION:
(TIME –DEPENDENT FORM)

- Substituting $E\psi$ and $P^2\ \psi$ from equation (6) and(5)
- Time dependent form of Schrodinger's equation
- $i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + \mathrm{U}\psi$ ....(9)
- In 3 dimensions
- $i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) + \mathrm{U}\psi$ ....(10)