COMPTON EFFECT

FOR

B.Sc. T.Y.

BY

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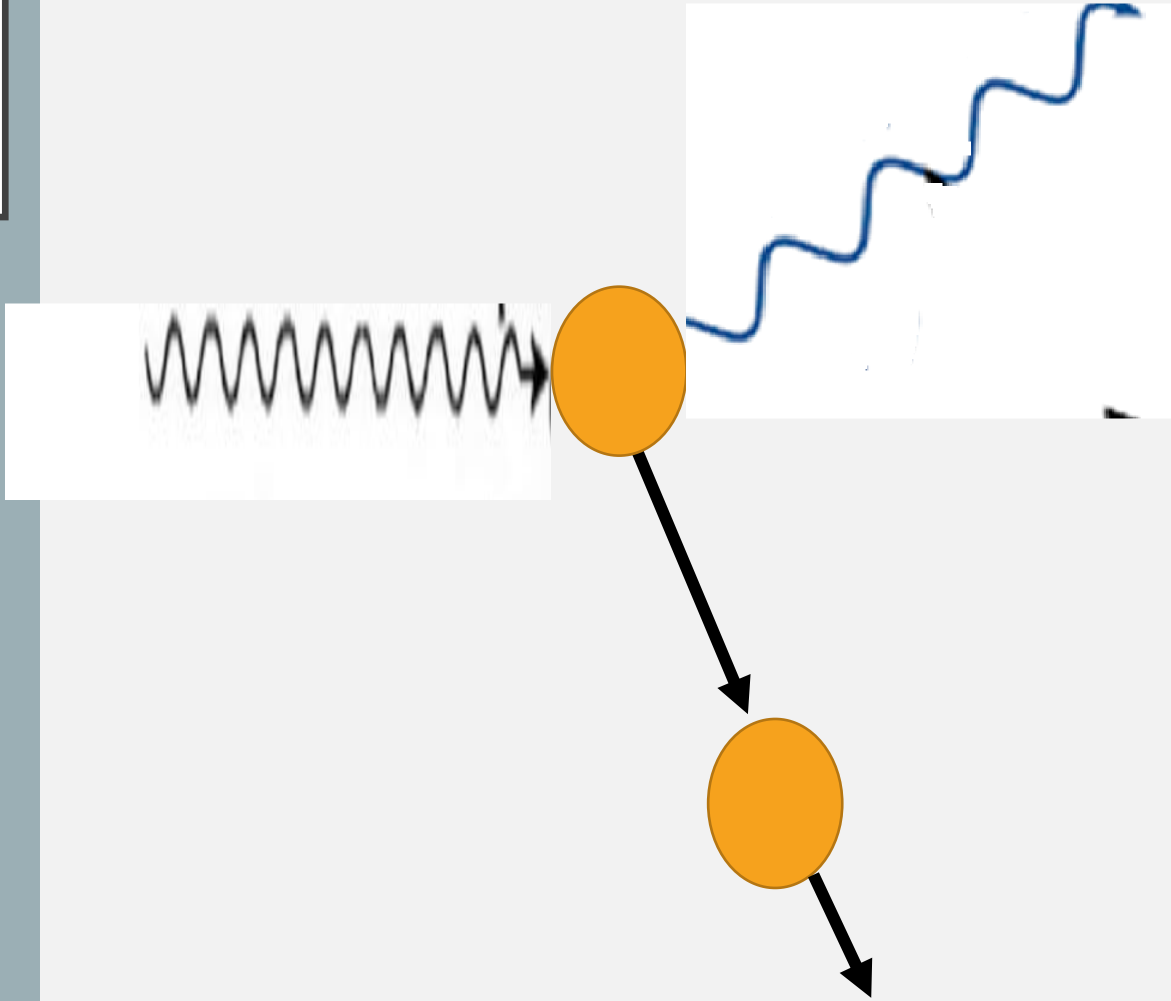
COMPTON EFFECT

Scattering of photon by an electron



ARTHUR HOLLY COMPTON
(1892-1962)

Native : Ohio

Education: Wooster and Princeton

Research: Washington university

X-ray increases in wavelength when scattered

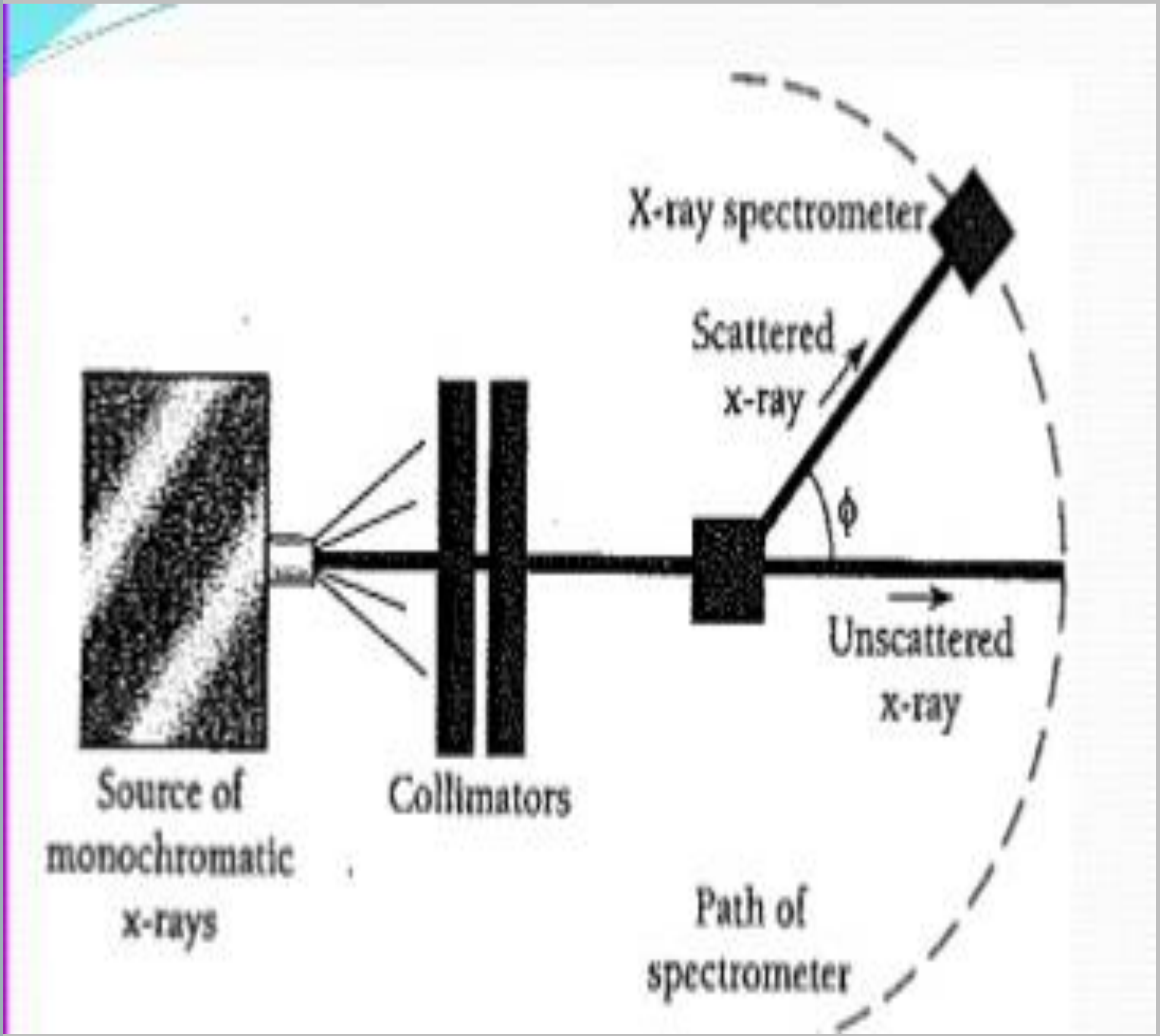
Conformation of particle nature of light

Studied cosmic rays

Nobel prize in 1927

EXPERIMENTAL DEMONSTRATION

- X –Ray of single wavelength is directed at target
- Wavelength of scattered X-Rays determined at various angle ϕ
- Result shows the shift in wavelength



COMPTON EFFECT

Scattering of photon by an electron

Suppose an X ray photon strikes on electron

Scattered photon changes its direction

Electron receives impulse begins to move

Before Collision

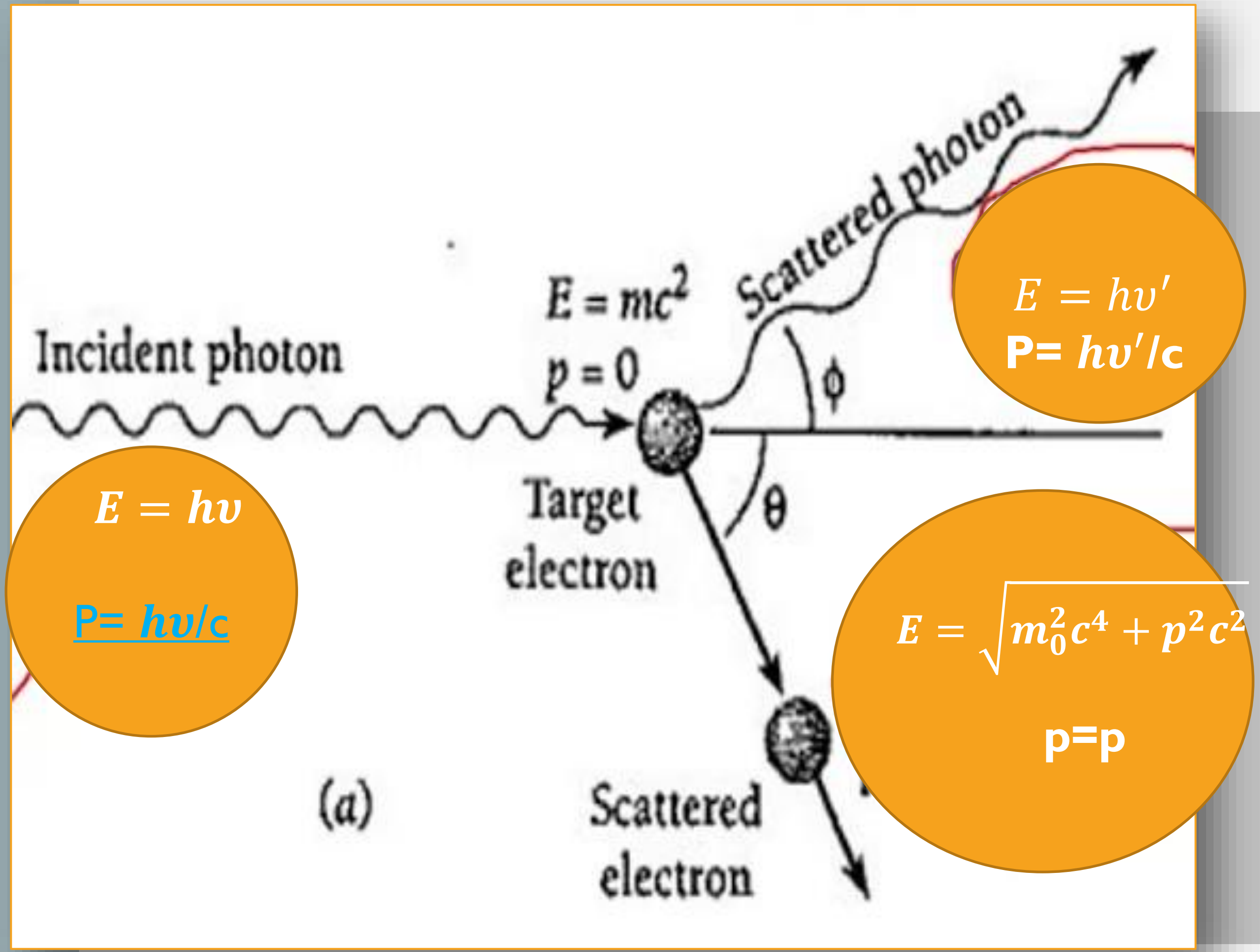
Energy associated with initial photon of frequency υ.is

$$E = h\upsilon$$

After collision

Energy associated with scattered photon of frequency $\upsilon'$is

$$E = h\upsilon'$$



COMPTON EFFECT

During collision

Photon looses energy

Electron receives that one

Loss in photon energy=Gain of KE of electron

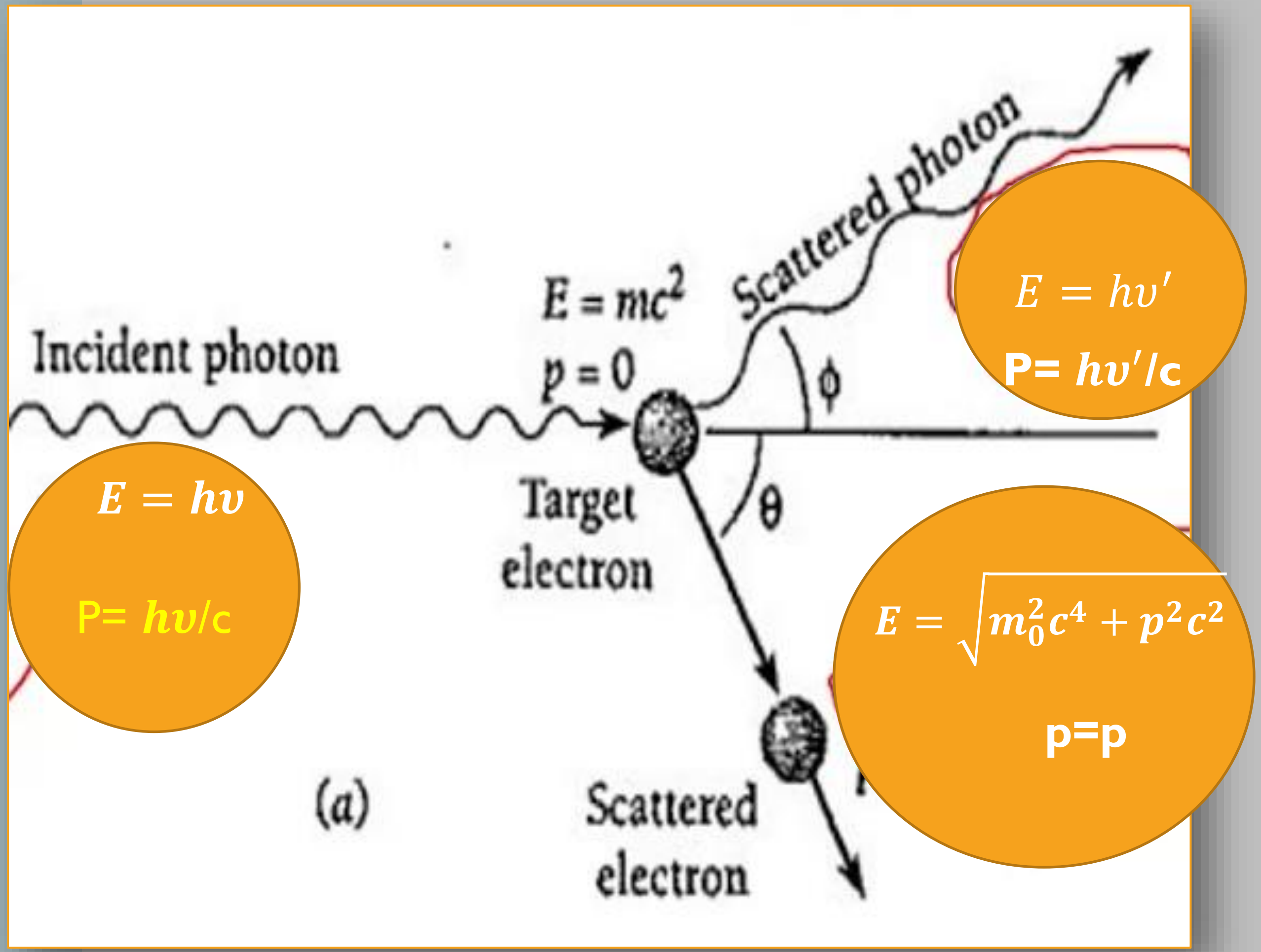
$h\upsilon - h\upsilon'$= KE -------------(1)

Momentum of massless particle related to its energy by

E=pc ------------------------------(2)

Photon momentum

p=E/c= $h\upsilon$/c -------------------(3)



COMPTON EFFECT

Momentum is conserved in each two mutually perpendicular direction

Initial photon momentum $= h\upsilon/c$

Scattered photon momentum $= h\upsilon'/c$

Initial electron momentum= 0

Final electron momentum = p

In original direction

Initial momentum=final momentum

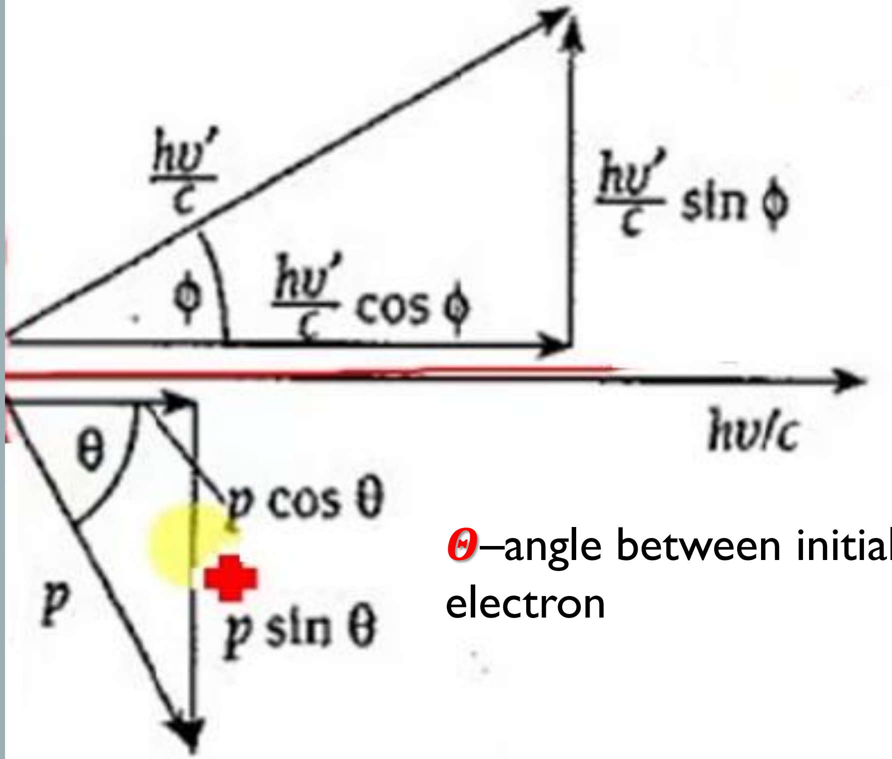
$$\frac{h\upsilon}{c} + 0 = \frac{h\upsilon'}{c}\cos\phi + p\cos\theta \text{-----(4)}$$

Perpendicular direction

Initial momentum=final momentum

$$0 = \frac{h\upsilon'}{c}\sin\phi + p\sin\theta \text{--------------(5)}$$

ϕ –angle between initial and final photon



θ–angle between initial photon and recoil electron

COMPTON EFFECT

Multiplying eq.(4) and (5) by c and rewriting

$pc\ cos\theta = h\upsilon - h\upsilon' cos\phi$ ----(6)

$pc\ sin\theta = h\upsilon' sin\phi$ ----- ---(7)

Squaring and adding eq. (6) and (7) θ is eliminated

$p^2c^2 = (h\upsilon)^2$-2$(h\upsilon)(h\upsilon')$cosϕ+$(h\upsilon')^2$------(8)

Total energy of particle is

E = (K.E.)+m_0c^2 -------(9)

$$\boldsymbol{E = \sqrt{m_0^2c^4 + p^2c^2}}$$ ------(10)

From (9) and(10)

$(K.E. + m_0c^2)^2$ =$m_0^2c^4 + p^2\ c^2$

$p^2c^2 = (K.E.)^2$+2$m_0c^2(K.E)$

But KE= $h\upsilon - h\upsilon'$

COMPTON EFFECT

$p^2c^2 = (hv)^2$-$2(hv)(hv')$ +$(hv')^2$+ $2m_0c^2(hv - hv')$-----(11)

Substituting p^2c^2in eq.(8)

$2m_0c^2(hv - hv') = 2(hv)(hv')(1-\cos\phi)$ -------------------(12)

Dividing eq. (12) by $2\ h^2c^2$

$\frac{m_0c}{h}(\frac{v}{c} - \frac{v'}{c}) = \frac{v}{c} - \frac{v'}{c}(1-\cos\phi)$

Since $\frac{v}{c} = \frac{1}{\lambda}$ *and* $\frac{\upsilon'}{c} = \frac{1}{\lambda'}$

$\frac{m_0c}{h}(\frac{1}{\lambda} - \frac{1}{\lambda'}) = \frac{1-cos\phi}{\lambda\lambda'}$

COMPTON EFFECT

$\lambda' - \lambda = \frac{h}{m_0c}(1-\cos\phi)$ ---------------(13)

$\frac{h}{m_0c} = \lambda_c$ Is Compton Wavelength

CONCLUSION:

1) Compton wavelength gives the scale of wavelength change of incident photon

2) Greatest wavelength change occurs when ϕ =180 degree which is twice λ_c

3) Maximum wavelength observed in X-Rays for visible it is less than 0.01%

4) X-Rays lose energy when they pass through the matter

5) Compton effect gives conformation to photon model

SUMMARY:

Compton effect is scattering of photon by electron

Energy associated with initial photon of frequency $\boldsymbol{\upsilon}$.is $\boldsymbol{E = h\upsilon}$

Energy associated with scattered photon of frequency $\upsilon'$is $\boldsymbol{E = h\upsilon'}$

Loss in photon energy=Gain of KE of electron

$\boldsymbol{h\upsilon - h\upsilon'}$**= KE**

Momentum is conserved in each two mutually perpendicular direction

COMPTON EFFECT

λ'- λ = $\frac{h}{m_0 c}$ $(1-\cos\phi)$ ---------------(13)

$\frac{h}{m_0 c}$= λ_c Is Compton Wavelength =2.426 pico metre

NUMERICAL PROBLEMS:

X-ray of wavelength 10.0pm are scarred from a target.

a)Find the wavelength of Xray scattered through 45^0

b) Find maximum wavelength present in the scattered X rays

c) Find maximum kinetic energy of recoil electron

Solution:

Given: Wave length of incident X-ray $=\lambda$ = 10 pm=10.0×10^{-12}m

$\frac{h}{m_0 c} = \lambda_c$ Is Compton Wavelength =2.426pm

Scattered angle = ϕ = 45^0 ,Wave length of scattered X-ray = λ'=?

Maximum Wave length of scattered X-ray = $\lambda' - \lambda$ =?

Maximum kinetic energy of recoil electron=?

a) Compton effect

$$\lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\phi) = \lambda' - \lambda = \lambda_c(1-\cos\phi)$$

$\lambda' = \lambda + \lambda_c(1-\cos\phi)$ substituting the values

λ' = 10pm+2.426pm(1-cos45^0)

= 10 pm+2.426pm(1-.7071)=10pm+0.71pm

= Wave length of scattered X-ray = λ'=10.7pm

NUMERICAL PROBLEMS:

b) Maximum Wave length of scattered X-ray = λ' - λ

when cos ϕ=180^0 i.e. $(1-\cos\phi)$= 2

$\lambda' = \lambda + 2\lambda_c$=10pm+2x 2.426pm

= 10pm+ 4. 852pm=14.852pm

Maximum Wave length of scattered X-ray = λ' - λ =14.852pm

c) Maximum kinetic energy of recoil electron=Difference in energies of incidence and scattered photons

K.E.$_{max}$= h$(\upsilon - \upsilon')$ =hc $(\frac{1}{\lambda} - \frac{1}{\lambda'})$

$=(6.63x10 - 34x3x108 \left(\frac{1}{10.0pm} - \frac{1}{14.9pm}\right)$

= 6.54x10^{-15} joule

$= \frac{6.54x10-15}{1.6x10-19}$ =4.08 KeV

$P = h\upsilon / c$

$$E = mc^2$$

$$E = mc.c$$

$$E = pc$$

$$p = \frac{E}{c}$$

$$p = \frac{h\upsilon}{c}$$

$$E = \sqrt{m_0^2 c^4 + p^2 c^2}$$

Relativistic total energy=E=$mc^2 = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$

Relativistic momentum=p=$mv = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}$

$E^2 = \frac{m_0^2 c^4}{1-\frac{v^2}{c^2}}$ and $p^2 = \frac{m_0^2 v^2}{1-\frac{v^2}{c^2}}$

$$E^2 - p^2c^2 = \frac{m_0^2 c^4 - m_0^2 v^2 c^2}{1-\frac{v^2}{c^2}}$$

$$= \frac{m_0^2 c^4 (1-\frac{v^2}{c^2})}{(1-\frac{v^2}{c^2})} = m_0^2 c^4$$

$$E^2 = m_0^2 c^4 + p^2 c^2$$

$$E = \sqrt{m_0^2 c^4 + p^2 c^2}$$