HEISENBERG'S UNCERTAINTY PRINCIPLE

STATEMENT:

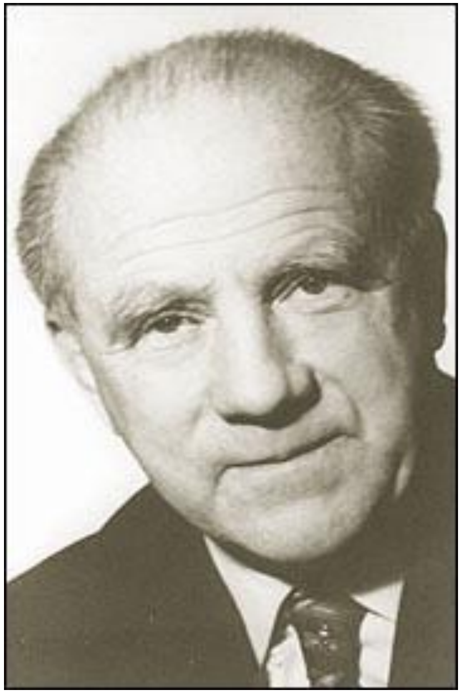
Uncertainty principle states that the product of uncertainty Δx in the position of an object at same instant and uncertainty Δp in its momentum components in the x direction at the same instant is equal to or greater than h/4π or ħ /2

i.e.

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}$$

Heisenberg’s uncertainty principle (Nobel Prize,1932)

- WERNER HEISENBERG (1901 - 1976)
- was one of the greatest physicists of the twentieth century. He is best known as a founder of quantum mechanics, the new physics of the atomic world, and especially for the uncertainty principle in quantum theory. He is also known for his controversial role as a leader of Germany's nuclear fission research during World War II. After the war he was active in elementary particle physics and West German science policy.
- *http://www.aip.org/history/heisenberg/p01.htm*



A particle is represented by a wave packet/pulse

- Since we experimentally confirmed that particles are wave in nature at the quantum scale *h* (matter wave) we now have to describe particles in term of waves (relevant only at the quantum scale)
- Since a real particle is localised in space (not extending over an infinite extent in space), the wave representation of a particle has to be in the form of wave packet/wave pulse

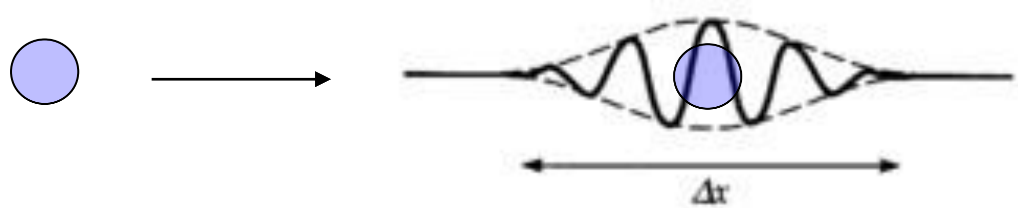


FIGURE 6.14 An idealized wave packet localized in space over a region Δx is the perposition of many waves of different amplitudes and frequencies.

Heisenberg uncertainty relations

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}$$

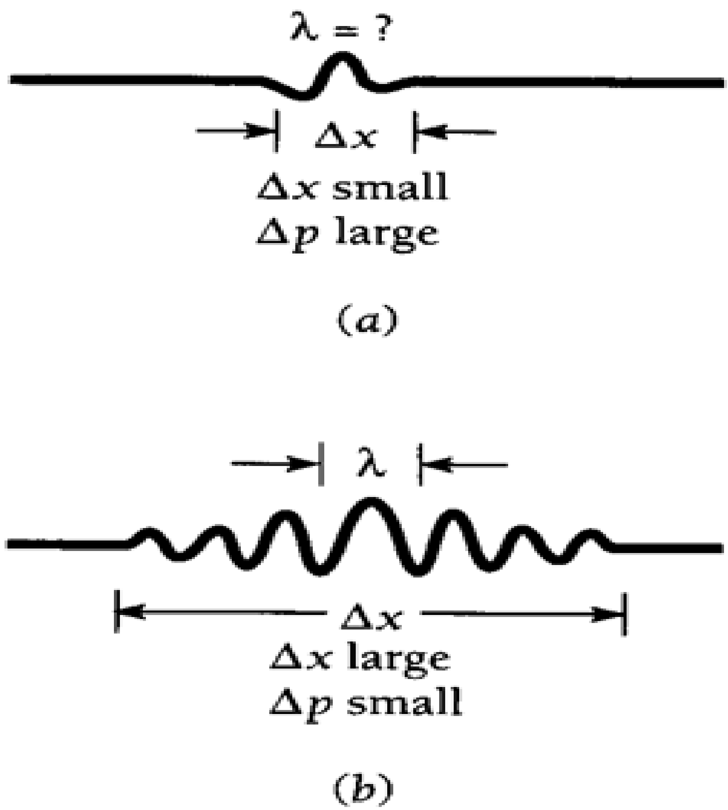


Figure 3.12 (*a*) A narrow de Broglie wave group. The position of the particle can be precisely determined, but the wavelength (and hence the particle's momentum) cannot be established because there are not enough waves to measure accurately. (*b*) A wide wave group. Now the wavelength can be precisely determined but not the position of the particle.

- **The Fourier integral can prove the following universal properties of all wave.** $\Delta x \Delta \kappa \geq 1/4\pi$ for $\kappa = 1/\lambda$, and $\Delta t \Delta \nu \geq 1/4\pi$

for matter wave:

$$p = h/\lambda \Rightarrow 1/\lambda = \kappa = p/h$$

$$\Delta x \Delta \kappa = \Delta x \Delta(p/h) = (1/h)\Delta x \Delta p \geq 1/4\pi$$

$$\Rightarrow \Delta p \Delta x \geq \hbar/2$$

uncertainty principle

$$E = h\nu \Rightarrow \nu = E/h \Rightarrow \Delta t \Delta(E/h) = (1/h)\Delta t \Delta E$$

$$\Rightarrow \Delta E \Delta t \geq \hbar/2$$

uncertainty principle

the consequence of duality

- Because…we are too large and quantum effects are too small

- Consider two extreme cases:
- (i) an electron with kinetic energy K = 54 eV, de Broglie wavelength, $\lambda = h/p = h / (2m_e K)^{1/2}$ = 1.65 Angstrom.
- Such a wavelength is comparable to the size of atomic lattice, and is experimentally detectable

- (ii) As a comparison, consider an macroscopic object, a billard ball of mass m = 100 g moving with momentum p
- $p = mv \approx 0.1$ kg $\times$ 10 m/s = 1 Ns (relativistic correction is negligible)
- It has de Broglie wavelength $\lambda = h/p \approx 10^{-34}$ m, too tiny to be observed in any experiments
- The total energy of the billard ball is
  - $E = K + m_0c^2 \approx m_0c^2 = 0.1\times (3 \times 10^8)^2$ J $= 9 \times 10^{15}$J
- (K is ignored since $K \ll m_0c^2$)
- The frequency of the de Broglie wave associated with the billard ball is
  $f = E/h = m_0c^2/h = (9\times10^{15}/6.63\times10^{34})$ Hz $=10^{78}$ Hz, impossibly high for any experiment to detect

Applications of uncertainty principle:

- 1) Electron does not exist in nucleus:
- The nucleus radius is of the order of 10^{-14} m.If we assume electron is confined in a nucleus the uncertainty in its position is $\Delta x = 2 \times 10^{-14}$ equal to diameter
- $\hbar = 6.63 \times 10^{-34} / 6.28 = 1.05 \times 10^{-34}$ Js
- Uncertainty in Δp is

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}$$

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- $\Delta p = 2.6 \times 10^{-21}$ The energy of electron =

Chapter 3 de Broglie's postulate: wavelike properties of particles

Ex: An atom can radiate at any time after it is excited. It is found that in a typical case the average excited atom has a life-time of about 10^{-8} sec. That is, during this period it emit a photon and is deexcited. (a) What is the minimum uncertainty $\Delta\nu$ in the frequency of the photon? (b) Most photons from sodium atoms are in two spectral lines at about $\lambda = 5890\,\overset{o}{A}$. What is the fractional width of either line, $\Delta\nu/\nu = ?$ (c) Calculate the uncertainty ΔE in the energy of the excited state of the atom. (d) From the previous results determine, to within an accuracy ΔE, the energy E of the excited state of a sodium atom, relative to its lowest energy state, that emits a photon whose wavelength is centered at $5890\,\overset{o}{A}$

$$\text{(a)}\ \Delta\nu\Delta t \ge 1/4\pi \Rightarrow \Delta\nu \ge 1/4\pi\Delta t = 8\times10^{-6}\ \text{sec}^{-1}$$

$$\text{(b)}\ \nu = c/\lambda = 3\times10^{10}/5890\times10^{-8} = 5.1\times10^{14}\ \text{sec}^{-1}$$

$$\Rightarrow \Delta\nu/\nu = 8\times10^{6}/5.1\times10^{14} = 1.6\times10^{-8}\ \text{natural width of the spectral line}$$

$$\text{(c)}\ \Delta E \ge \frac{h/4\pi}{\Delta t} = \frac{h}{4\pi\Delta t} = \frac{6.63\times10^{-34}}{4\pi\times10^{-8}} \approx 3.3\times10^{-8}\ \text{eV}\quad \text{the width of the state}$$

$$\text{(d)}\ \Delta\nu/\nu = \text{h}\Delta\nu/\text{h}\nu = \Delta E/E \Rightarrow E = \Delta E/(\Delta\nu/\nu) = 2.1\,\text{eV}$$

Time-energy uncertainty

- Just as $\Delta p_x \Delta x \geq \frac{\hbar}{2}$ implies position-momentum uncertainty relation, the classical wave uncertainty relation $\Delta \nu \Delta t \geq \frac{1}{4\pi}$ also implies a corresponding relation between time and energy

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

- This uncertainty relation can be easily obtained:

$$h \Delta \nu \Delta t \geq \frac{h}{4\pi} = \frac{\hbar}{2};$$

$$\because E = h\nu, \Delta E = h\Delta \nu \Rightarrow \Delta E \Delta t = h \Delta \nu \Delta t = \frac{\hbar}{2}$$

What $\Delta p_x \Delta x \geq \frac{\hbar}{2}$ means

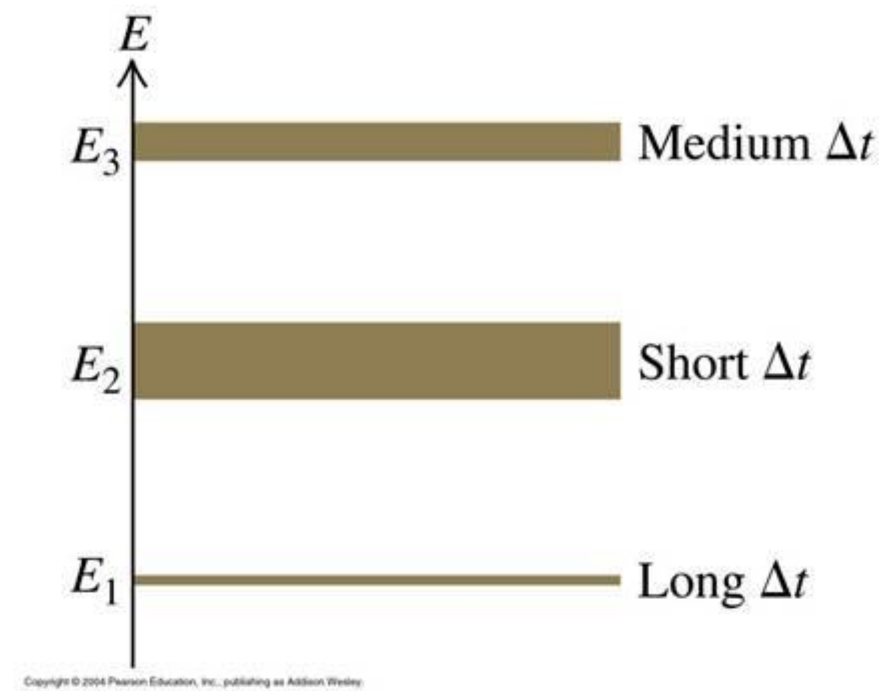
- It sets the intrinsic lowest possible limits on the uncertainties in knowing the values of p_x and x, no matter how good an experiments is made
- It is impossible to specify simultaneously and with infinite precision the linear momentum and the corresponding position of a particle

What $\Delta E \Delta t \geq \frac{\hbar}{2}$ means

- Uncertainty principle for energy.
- The energy of a system also has inherent uncertainty, ΔE
- ΔE *is* dependent on the *time interval* Δt during which the system remains in the given states.
- If a system is known to exist in a state of energy E over a limited period Δt, then this energy is uncertain by at least an amount $h/(4\pi\Delta t)$. This corresponds to the 'spread' in energy of that state (see next page)
- Therefore, the energy of an object or system can be measured with infinite precision (ΔE=0) only if the object of system exists for an infinite time ($\Delta t \rightarrow \infty$)

What $\Delta E \Delta t \geq \frac{\hbar}{2}$ means

- A system that remains in a metastable state for a very long time (large Δt) can have a very well-defined energy (small ΔE), but if remain in a state for only a short time (small Δt), the uncertainty in energy must be correspondingly greater (large ΔE).



Example

- A typical atomic nucleus is about 5.0×10^{-15} m in radius. Use the uncertainty principle to place a lower limit on the energy an electron must have if it is to be part of a nucleus