UNIT -1
WAVES

B.Sc. Second Year (PHYSICS)

By

Bhanudas narwade

INTRODUCTION:

Oscillatory motion:

Motion that moves to and fro about mean position

Oscillator:

Body that oscillates

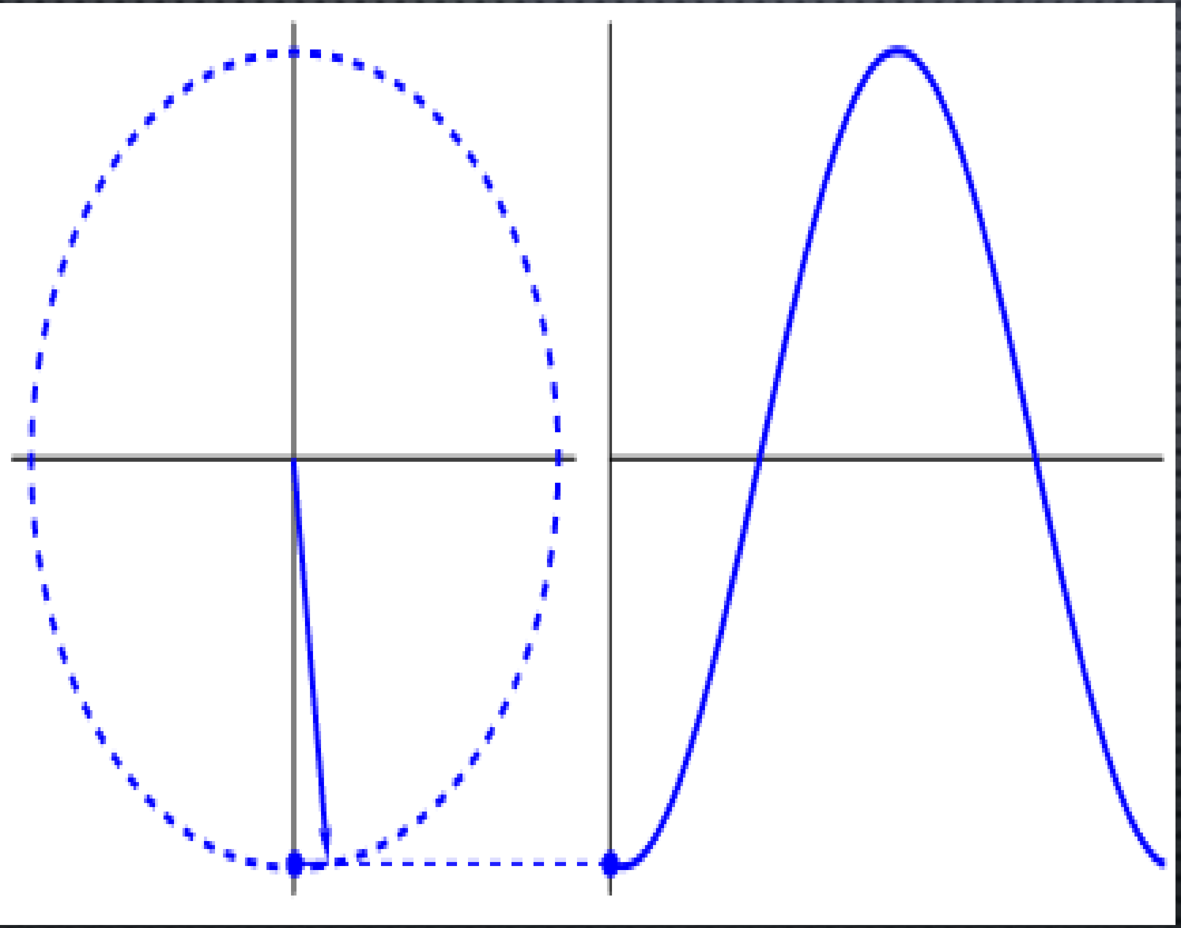
Examples of oscillatory motion:

1) Motion of pendulum in a clock.
2) Motion of vibrating string
3) Motion of prongs of tuning fork
4) Motion of needle of sewing machine

Periodic Motion:

Motion that repeats itself after equal interval of time

WAVES



INTRODUCTION:

Period:

Time taken to complete one oscillation

Frequency:

Number of oscillations performed by body in unit time

Waves:

An Oscillatory disturbance travelling through medium without of change of form

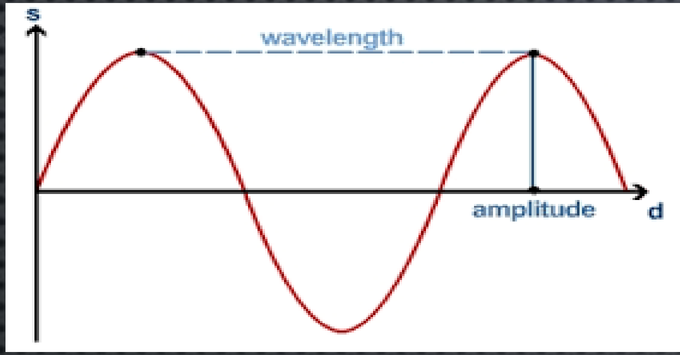
Wave Motion:

Mode of transfer of energy
from one point to another
Through material medium
In the form of oscillatory disturbance

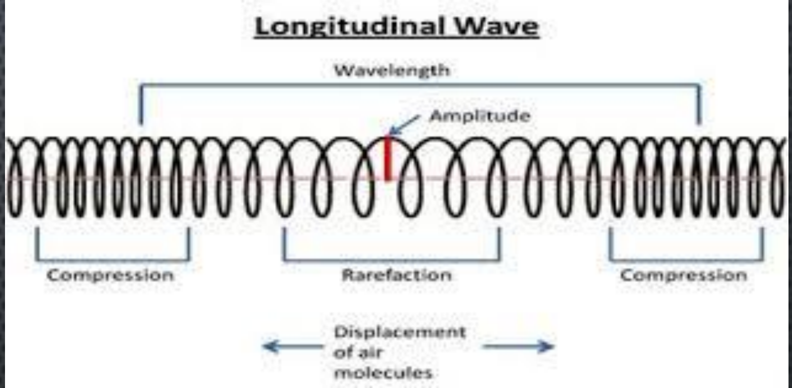
Wave motion transfer energy not matter

Progressive waves:

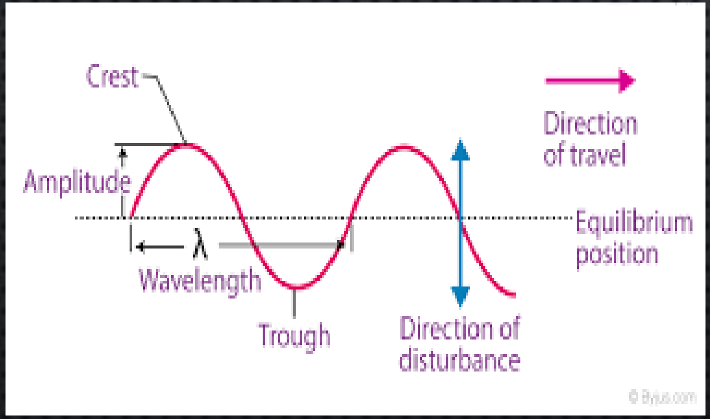
Waves continuously travels in specific direction



Longitudinal waves:



Transverse waves:



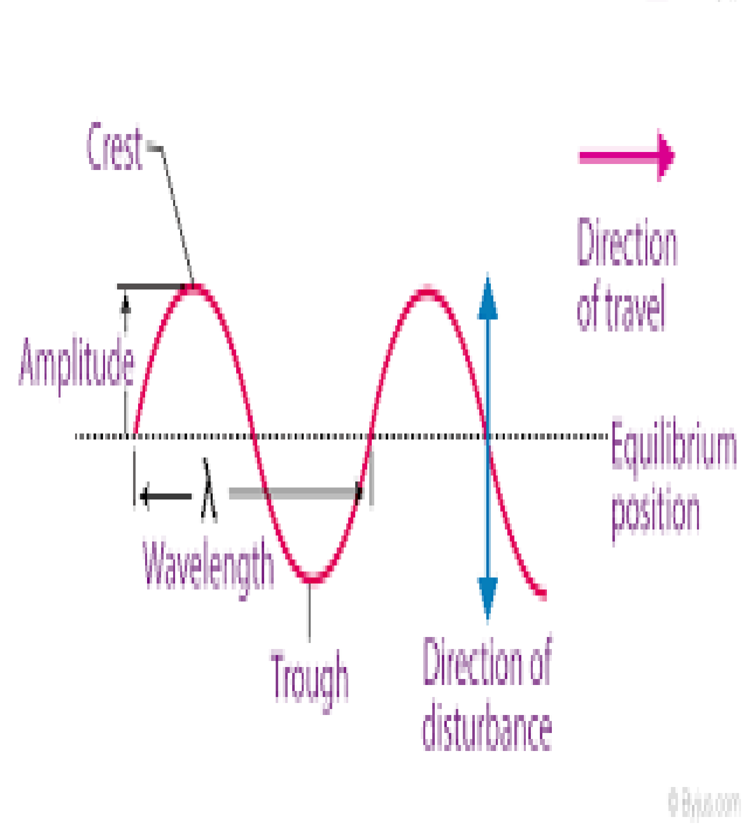
WAVES:

Characteristics of waves

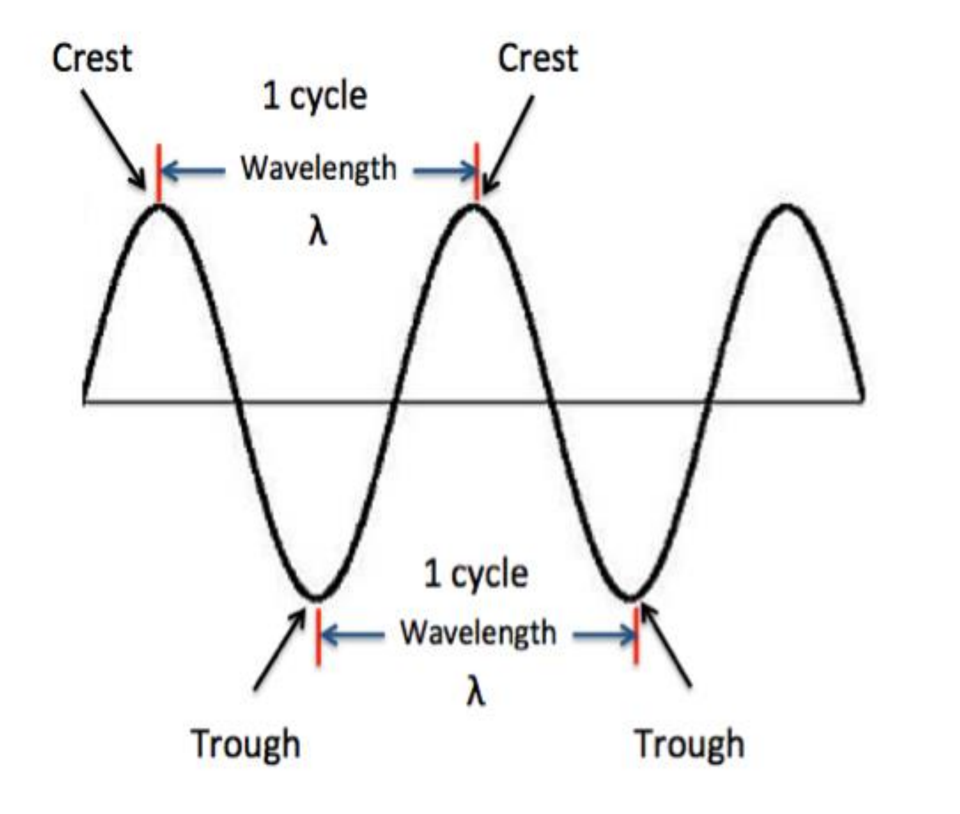
Amplitude(A):
Maximum displacement of particle from mean position on both sides
SI unit : meter

Wavelength (λ):
Distance between two successive particles which are in the same state of vibration
SI Unit :meter

Frequency (n):
Number of oscillations per unit time
SI Unit: hertz (Hz)



RELATION BETWEEN WAVE VELOCITY ,WAVELENGTH AND FREQUENCY



Crest, Trough and Wavelength

Wave velocity(v):

Distance covered by oscillatory disturbance per second.

Wave velocity

$$(v)= \frac{Distance\ covered\ by\ wave\ in\ one\ oscillation}{Time\ required}$$

$$V=\frac{wavelangth}{Time}= \frac{\lambda}{T}$$ But $\frac{1}{T}$ = Frequency(n)

$$V = n\lambda$$

Wave velocity= wave length x Frequency

NUMERICAL PROBLEMS:

Prob.1: A sounding source sends out waves of length 1.5m in air. If the velocity of sound in air is 330m/s ,what is the frequency of sounding source?

SOLUTION:

given data :

velocity of waves v =330m/s

Wavelength of waves λ =1.5m

Frequency of source n=?

Formula: $\mathbf{v} = n\lambda$

$$\therefore \quad n = \frac{v}{\lambda}$$

$$= \frac{330}{1.5} = 220\text{Hz}$$

Frequency of sounding source n=220Hz

NUMERICAL PROBLEMS:

Prob.2: The frequency of vibrating tuning fork is 480 Hz and velocity of sound in air is 320m/s. How far have sound waves reached when tuning fork has completed 120 vibrations?

SOLUTION:

given data : velocity of waves $v = 320 m/s$

Wavelength of waves $\lambda = ?$

Frequency of source $n = 480$ Hz

Distance covered by wave in 120 vibrations=?

Formula: $v = n\lambda$

$$\therefore \lambda = \frac{v}{n} = \frac{320}{480} = 0.66m$$

In one vibration wave covers a distance= $\lambda = 0.66m$

$\therefore$ In 120 vibration wave covers = 120x **0.66** =80m

SIMPLE HARMONIC PROGRESSIVE WAVES:

Displacement of particle "o" in its origin is

$$y = asin\omega t \qquad ----(1)$$

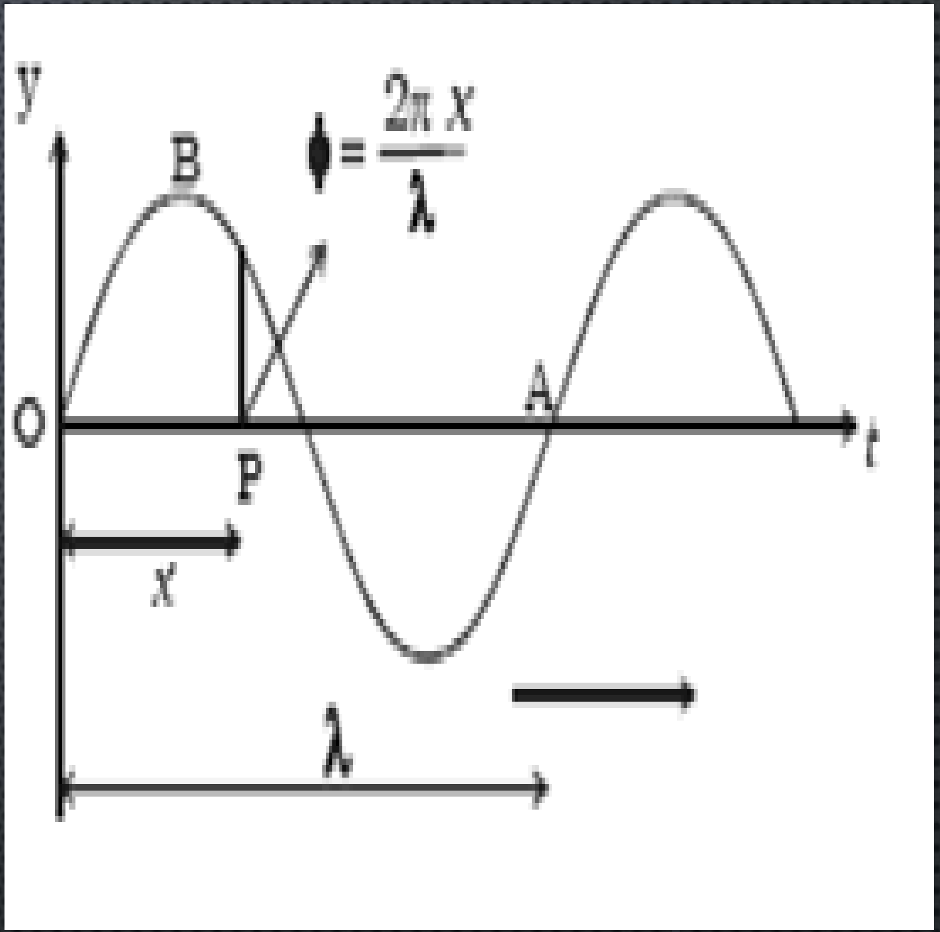
The displacement at the same instant of particle "P" at a distance "x" from "o" is

$$y = asin\omega t - \phi \text{ -----(2)}$$

ϕ is phase lag $\qquad \phi = \frac{2\pi x}{\lambda}$

equation (2) becomes

$$y = a\,sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right)$$



EQUATION OF SIMPLE HARMONIC PROGRESSIVE WAVES:

But $V = n\lambda$ or $V = \frac{\lambda}{T}$ or $\frac{1}{T} = \frac{V}{\lambda}$

$y = a\, sin \frac{2\pi}{\lambda}(Vt - x)$ ----(3)

Wave travelling in negative direction of X axis

$y = a\, sin \frac{2\pi}{\lambda}(Vt + x)$

NUMERICAL PROBLEM:

Prob. 1: When a simple harmonic progressive wave is propagated through a medium, the displacement of a particle in cm at any instant of time is given by

$$y = 10\sin\frac{2\pi}{100}(36000t - 20)$$

Calculate i) Amplitude of the vibrating particle ii) Wave velocity iii) Wavelength and iv) time period

Solution:

The given equation of simple harmonic progressive wave is

$$y = 10\sin\frac{2\pi}{100}(36000t - 20) \quad ----(1)$$

The standard equation of simple harmonic progressive wave is

$$y = a\,sin\frac{2\pi}{\lambda}(Vt - x) \quad ---- \quad (2)$$

NUMERICAL PROBLEM:

$$y = 10\sin\frac{2\pi}{100}(36000t - 20) \quad \text{----(1)}$$

$$y = a\,sin\frac{2\pi}{\lambda}(Vt - x) \quad \text{----} \qquad (2)$$

Comparing equations (1) and (2)

i) Amplitude of vibrating particle= a=10 cm

ii) Wavelength of wave = λ=100 cm

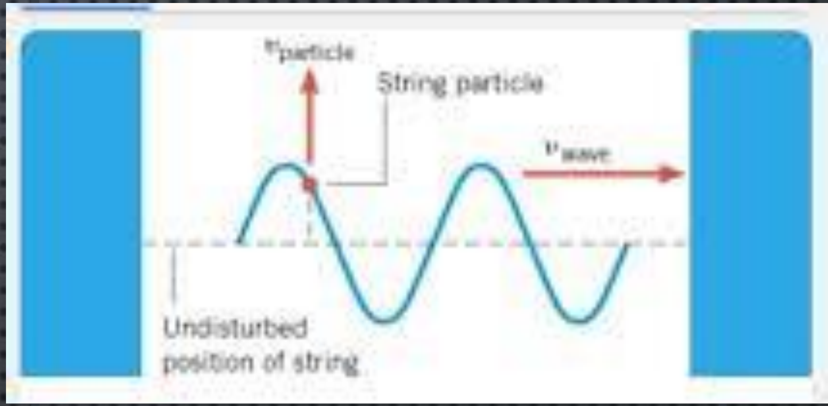
iii) Wave velocity =V= 36000cm/s

iv) Frequency =n=$\frac{V}{\lambda}$ = $\frac{36000}{100}$ = 360 Hz

v) Time period =T= $\frac{1}{n}$ = $\frac{1}{360}$ = 0.00277 sec.

WAVE VELOCITY AND PARTICLE VELOCITY:

Wave velocity is the distance covered by oscillatory disturbance per unit time in a medium
Velocity of transmission of wave



Particle velocity is the velocity of vibrating particle about mean position.
Velocity of particles vibration

RELATION BETWEEN PARTICLE VELOCITY AND WAVE VELOCITY:

The standard equation of simple harmonic progressive wave is

$$y = a\, sin\frac{2\pi}{\lambda}(vt - x) \text{ ---- (1)}$$

Particle velocity is U= $\frac{dy}{dt}$

Differentiating equation (1) wrt "*t*"

$$U = \frac{dy}{dt} = \frac{2\pi a v}{\lambda} \cos\frac{2\pi}{\lambda}(vt - x) \text{-----(2)}$$

Maximum value of particle velocity is

$$U_{max} = \frac{2\pi a v}{\lambda} \qquad \text{---- (3)}$$

RELATION BETWEEN PARTICLE VELOCITY AND WAVE VELOCITY:

U_{max}= $\frac{2\pi a v}{\lambda}$ =$\frac{2\pi a}{\lambda}$(wave velocity)

Particle Acceleration is $accn = \frac{d^2y}{dt^2}$

$$accn = -\frac{4\pi^2 a v^2}{\lambda^2}\left[sin\frac{2\pi}{\lambda}(vt - x)\right]$$ ----(4)

Rewriting $accn = -\frac{4\pi^2 v^2}{\lambda^2}\left[asin\frac{2\pi}{\lambda}(vt - x)\right]$

$$accn = -(\frac{4\pi^2 v^2}{\lambda^2})\ y$$

Acceleration is maximum when y=a

$$\therefore accn_{max.} = -\left(\frac{4\pi^2 v^2}{\lambda^2}\right)a$$ -- ---(5)

RELATION BETWEEN PARTICLE VELOCITY AND WAVE VELOCITY:

$negative\ sign\ indicates\ accn\ is\ directed\ towards\ mean\ position$

Differentiating equation (1) wrt "x"

$\frac{dy}{dx}$ = -$\frac{2\pi x}{\lambda}$ cos $\frac{2\pi}{\lambda}(vt - x)$ -----(6)

$\frac{dy}{dx}$ is slope of displacement curve

From equation (2) and (6) we have

$U = \frac{dy}{dt} = -v(\frac{dy}{dx})$ --------(7)

∴*Particle velocity*

at any instant =wave velocity x slope of dis.curve

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UNIT -1

WAVES

DIFFERNTIAL EQUATION OF WAVE MOTION

B.Sc. Second Year (PHYSICS)

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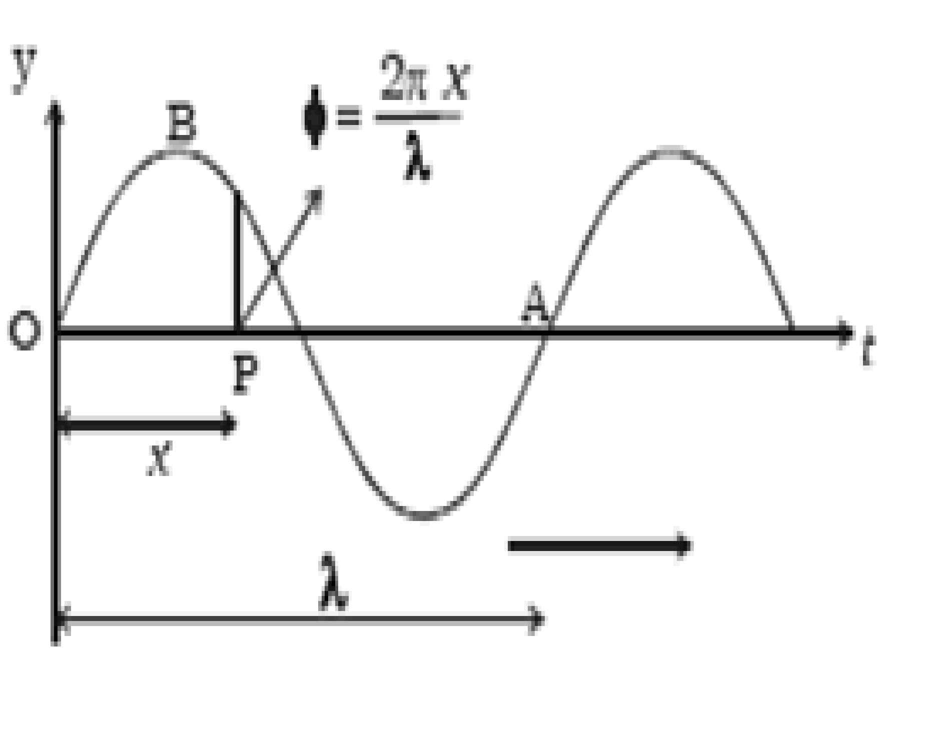
WHAT WE HAVE LEARN :

Waves
Wave motion
Characteristics of waves
Amplitude, wavelength, frequency, wave velocity and period
Progressive waves
Transverse waves
Longitudinal waves
Relation between wave velocity, frequency and wave length
Equation of SHPW
Wave velocity and particle velocity and relation

DIFFERNTIAL EQUATION OF WAVE MOTION



Consider a SHPW

The displacement (y) of particle at a distance x from origin at a instant of time (t) is

$$y = a\,sin\frac{2\pi}{\lambda}(vt - x) \text{ ---- (1)}$$

Where:

a is Amplitude

λ is Wavelength

v is Wave velocity

t is instantaneous time

x is position of particle from origin

DIFFERNTIAL EQUATION OF WAVE MOTION:

Particle velocity is U= $\frac{dy}{dt}$

Differentiating equation (1) wrt "t"

$\frac{dy}{dt}$= $\frac{2\pi av}{\lambda}$cos $\frac{2\pi}{\lambda}(vt - x)$-----(2)

Particle Acceleration is $accn = \frac{d^2y}{dt^2}$

Differentiating equation (2) wrt "t"

$\frac{d^2y}{dt^2} = -\frac{4\pi^2 av^2}{\lambda^2}\left[sin\frac{2\pi}{\lambda}(vt - x)\right]$----(3)

DIFFERNTIAL EQUATION OF WAVE MOTION:

Compression or strain is $\frac{dy}{dx}$

Differentiating equation (1) wrt "x"

$$\frac{dy}{dx} = -\frac{2\pi a}{\lambda} \cos \frac{2\pi}{\lambda}(vt - x) \text{ -----(4)}$$

Rate of change of Compression is $\frac{d^2y}{dx^2}$

Differentiating equation (4) wrt "x"

$$\frac{d^2y}{dx^2} = -\frac{4\pi^2 a}{\lambda^2}\left[sin \frac{2\pi}{\lambda}(vt - x)\right] \text{----(5)}$$

DIFFERNTIAL EQUATION OF WAVE MOTION:

From equation (2) and (4)

$$\frac{dy}{dt} = -v\frac{dy}{dx} \qquad \text{----(6)}$$

From equation (3) and (5)

$$\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2} \qquad \text{---(7)}$$

Equation (7) is a differential equation of wave motion

Important in mathematical Physics

Any equation in this form always represent a wave motion

WAY TOWARDS SOLUTION.......

$$y = a\,sin\frac{2\pi}{\lambda}(vt - x)$$

U= $\frac{dy}{dt}$

$$accn = \frac{d^2y}{dt^2}$$

Compression or strain $\frac{dy}{dx}$

Rate of change of Compression $\frac{d^2y}{dx^2}$

COMPARE .....

$$\frac{dy}{dt} = \frac{2\pi a v}{\lambda} \cos \frac{2\pi}{\lambda}(vt - x) \text{-----}(2)$$

$$\frac{dy}{dx} = -\frac{2\pi a}{\lambda} \cos \frac{2\pi}{\lambda}(vt - x) \text{-----}(4)$$

$$\frac{dy}{dt} = -v\frac{dy}{dx}$$

COMPARE .....

$$\frac{d^2y}{dt^2} = -\frac{4\pi^2 a v^2}{\lambda^2}\left[\sin\frac{2\pi}{\lambda}(vt - x)\right] \text{----(3)}$$

$$\frac{d^2y}{dx^2} = -\frac{4\pi^2 a}{\lambda^2}\left[\sin\frac{2\pi}{\lambda}(vt - x)\right] \text{----(5)}$$

$$\frac{d^2y}{dt^2} = v^2\,\frac{d^2y}{dx^2} \qquad \text{---(7)}$$

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ENERGY OF A PROGRESSIVE WAVE

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WHAT WE HAVE LEARN :

Equation of SHPW

Wave velocity and particle velocity
and relation

Differential equation of wave motion

$$\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}$$

ENERGY OF A PROGRESSIVE WAVE:

In progressive wave energy is continuously transfer

This energy is supplied by source

Energy transferred per second is also corresponds to energy possessed by the particles in a length “v”

Energy of wave is partly kinetic and partly potential

ENERGY OF A PROGRESSIVE WAVE:

K.E. is due to velocity of vibrating particle

Velocity is maximum at mean position and zero at extreme position

K.E. is maximum at mean position and zero at extreme position

P.E. is due to displacement of particle from mean position

ENERGY OF A PROGRESSIVE WAVE:

P.E. is maximum at extreme position and minimum at mean position

In longitudinal wave motion compression and rarefactions are produced

Energy distribution is not uniform over the wave

Transfer of energy and not matter

ANALYTICAL TREATMENT:

Equation of simple harmonic progressive wave is

$$y = a\, sin \frac{2\pi}{\lambda}(vt - x) \text{ ---- (1)}$$

Particle velocity is U= $\frac{dy}{dt}$

Differentiating equation (1) wrt "*t*"

$$\frac{dy}{dt} = \frac{2\pi a v}{\lambda} \cos \frac{2\pi}{\lambda}(vt - x) \text{-----(2)}$$

ANALYTICAL TREATMENT:

Particle Acceleration is $accn = \frac{d^2y}{dt^2}$

Differentiating equation (2) wrt "*t*"

$$\frac{d^2y}{dt^2} = -\frac{4\pi^2 av^2}{\lambda^2}\left[\sin\frac{2\pi}{\lambda}(vt - x)\right]$$----(3)

ANALYTICAL TREATMENT:

Potential energy:

Work done for displacement dy

$$= F\,dy$$

If ρ density of medium

Work done per unit volume for dy

$$= \rho\left(\frac{4\pi^2 a v^2}{\lambda^2} sin\,\frac{2\pi}{\lambda}(vt - x)\right) dy$$

ANALYTICAL TREATMENT:

Total work done for displacement y is

$$= \int_0^y \rho\left(\frac{4\pi^2 a v^2}{\lambda^2} sin\ \frac{2\pi}{\lambda}(vt - x)\right) dy$$

But

$$y = a\ sin\frac{2\pi}{\lambda}(vt - x)$$

$\therefore$ Potential energy per unit volume

$$= \left(\frac{4\pi^2 \rho v^2}{\lambda^2}\right) \int_0^y y dy$$

$$= \frac{4\pi^2 \rho v^2}{2\lambda^2} y^2 \qquad = \frac{2\pi^2 \rho v^2}{\lambda^2} y^2$$

ANALYTICAL TREATMENT:

P. E per unit Volume $=\frac{2\pi^2\rho v^2}{\lambda^2} a^2 sin^2\left[\frac{2\pi}{\lambda}(vt-x)\right]$----(4)

K.E. per unit volume $=\frac{1}{2}\rho U^2$

$$=\frac{1}{2}\rho\left[\frac{2\pi a v}{\lambda} cos\frac{2\pi}{\lambda}(vt-x)\right]^2$$

$=\frac{2\pi^2\rho v^2 a^2}{\lambda^2} cos^2\left[\frac{2\pi}{\lambda}(vt-x)\right]$---(5)

ANALYTICAL TREATMENT:

Adding eq. (4) and (5)

Total energy per unit volume

$$=\frac{P.E.}{VOLUME}+\frac{K.E.}{VOLUME}$$

$$=\frac{2\pi^2\rho v^2 a^2}{\lambda^2}\left[sin^2\frac{2\pi}{\lambda}(vt-x)+cos^2\frac{2\pi}{\lambda}(vt-x)\right]$$

Total energy per unit volume $E=\frac{2\pi^2\rho v^2 a^2}{\lambda^2}$

$E=2\pi^2\rho n^2 a^2$ ------(6)

ANALYTICAL TREATMENT:

Av. K.E. per unit volume= $\pi^2 \rho n^2 a^2$

Av. P.E. per unit volume= $\pi^2 \rho n^2 a^2$

For unit area of cross section,
wave velocity $= v$ $\quad Volume = 1 \times v = v$

Energy transferred per unit area per sec. =

$$= E\,v = 2\pi^2 \rho v n^2 a^2$$

CONCLUSION:

1. P.E. and K.E. of every particle changes with time

2. Av. K.E. per unit volume and Av. P .E. per unit volume remains constant

3. Total energy per unit volume remains constant

4.Energy of progressive wave at any instant is half kinetic and half potential

ADDING K.E. PER UINT VOLUME AND P.E.PER UNIT VOLUME:

$$\frac{P.E.}{VOLUME} = \frac{2\pi^2\rho v^2}{\lambda^2} a^2 sin^2\left[\frac{2\pi}{\lambda}(vt-x)\right]\text{----(4)}$$

$$\frac{K.E.}{VOLUME} == \frac{2\pi^2\rho v^2 a^2}{\lambda^2} cos^2\left[\frac{2\pi}{\lambda}(vt-x)\right]\text{---(5)}$$

$$=\frac{2\pi^2\rho v^2 a^2}{\lambda^2}\left[sin^2\frac{2\pi}{\lambda}(vt-x) + cos^2\frac{2\pi}{\lambda}(vt-x)\right]$$

$$\frac{TOTAL\ ENERGY.}{VOLUME} = \underline{E} = \frac{2\pi^2\rho v^2 a^2}{\lambda^2}$$

WAY TOWARDS SOLUTION…….

$$y = a\,sin\frac{2\pi}{\lambda}(vt - x)$$

U= $\frac{dy}{dt}$

$$accn = \frac{d^2y}{dt^2}$$

To obtain potential energy of particle

Work done for small displacement dy = F dy

Total work done foe y and P.E. per unit volume

K.E. per unit volume =$\frac{1}{2}\rho U^2$

Total energy per unit volume =$\frac{P.E.}{VOLUME} + \frac{K.E.}{VOLUME}$

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UNIT -1

WAVES

EQUATION OF MOTION OF A VIBRATING STRING

B.Sc. Second Year (PHYSICS)

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WHAT WE HAVE LEARN :

1. P.E. and K.E. of every particle changes with time

2. Av. K.E. per unit volume and Av. P .E. per unit volume remains constant

3. Total energy per unit volume remains constant

4. Energy of progressive wave at any instant is half Kinetic and half potential

TRANSVERSE VIBRATION OF STRING

Stringed musical instruments:

Guitar

Violin

Piano

Guitar-Strings are **plucked**

Violin-Strings are **bowed**

Piano-Strings are **struck**

EQUATION OF MOTION OF A VIBRATING STRING:

Assumptions:

1. Length of string –greater than diameter

2 Perfectly uniform and flexible

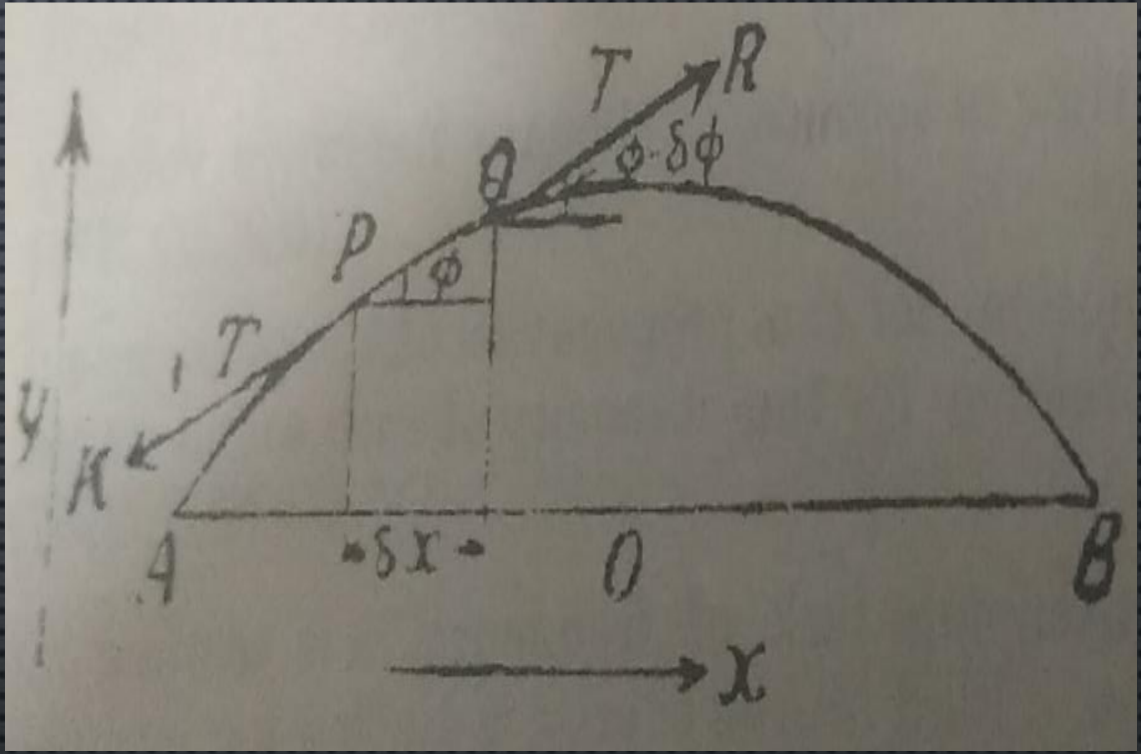
3 Stretched between two fixed points

4 Tension in string should be large

5 Effect of gravitational force can be neglected.

6. Tension T in string should be constant everywhere

EQUATION OF MOTION OF A VIBRATING STRING:



AB – stretched string with tension T

Plucked at Centre O

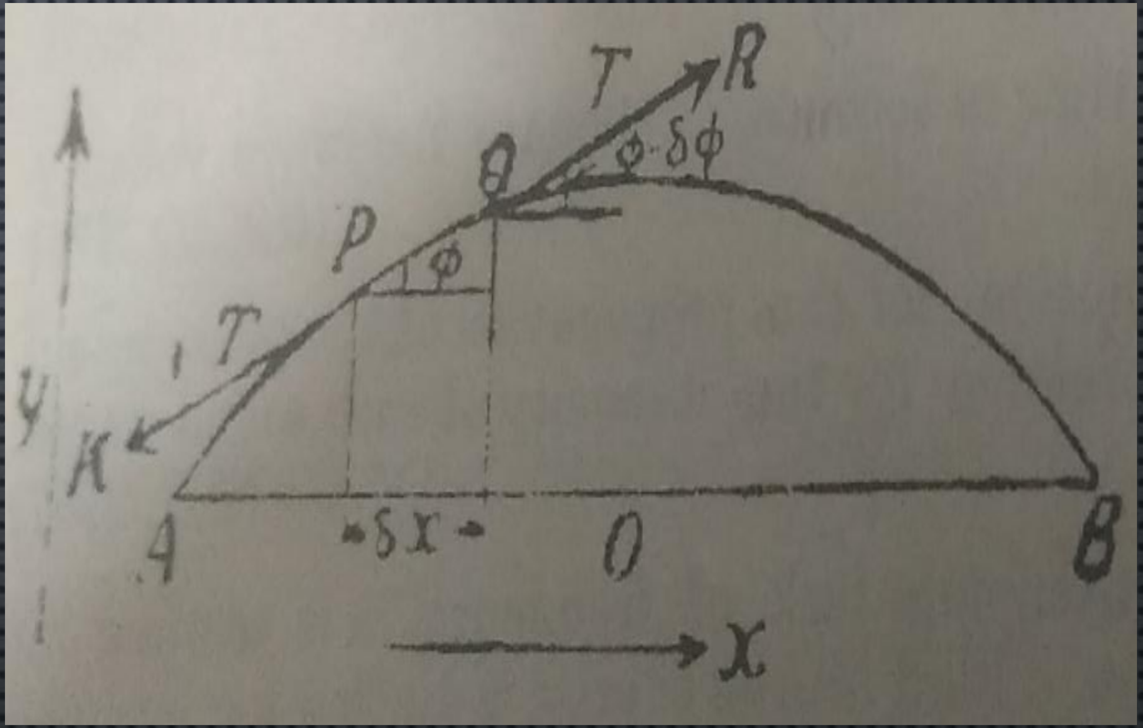
Set into transverse vibration

Under displaced condition

Displaced portion of string is very

small so T is constant everywhere

EQUATION OF MOTION OF A VIBRATING STRING:



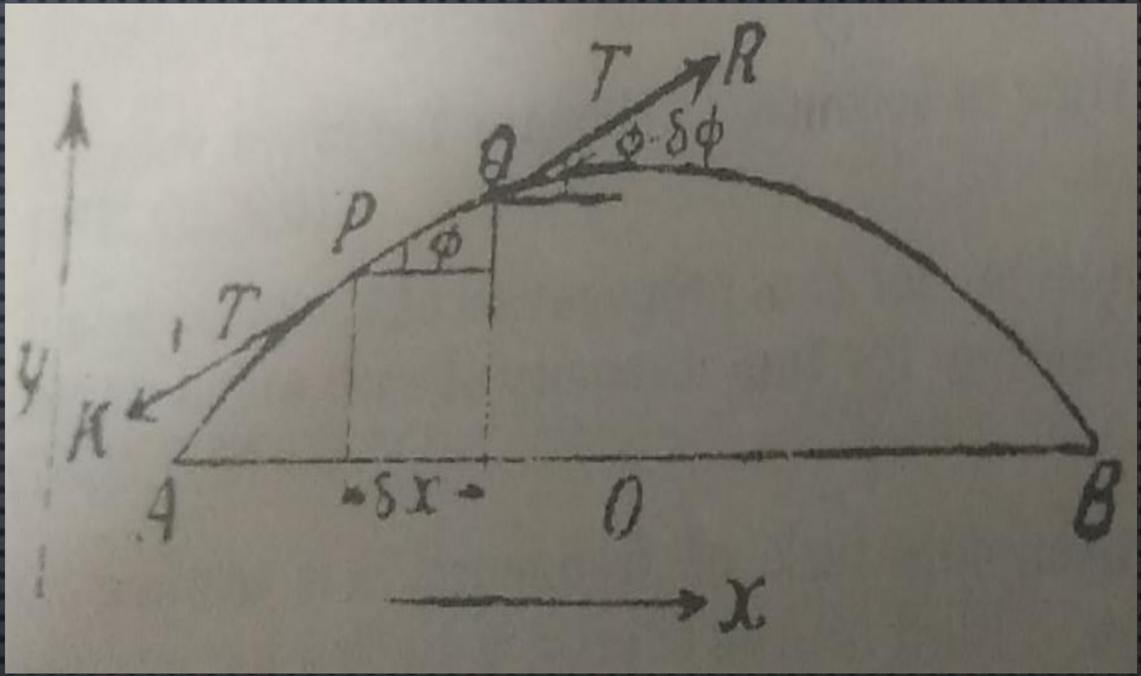
Motion is SHM- force ∝displacement

Undisplaced string is on X axis
Displacement of string ⊥ to length on Y axis

PQ- Small element of plucked string

δx– length of PQ
At PQ tension T acts along tangents PK and QR

EQUATION OF MOTION OF A VIBRATING STRING:



- ϕ And ϕ-$\delta\phi$ angle of inclination of tangents to curve at P and Q respectively of element δx

Resolving T into mutually ⊥ components

Horizontal components cancel out each other

EQUATION OF MOTION OF A VIBRATING STRING:

Resultant of vertical components in direction of Y axis is

$R = T sin\phi - T\ sin(\phi - \delta\phi)$------(1)

$$R = T[sin\phi - sin(\phi - \delta\phi)]$$

$$R = T\ cos\ \phi\delta\phi = T\ \delta(sin\phi)$$

For small ϕ, sinϕ=tanϕ

From fig.Tanϕ=$\frac{dy}{dx}$ slope of curve

EQUATION OF MOTION OF A VIBRATING STRING:

$$\therefore R = T\,\delta\left(\frac{dy}{dx}\right)$$

For distance δx

$$R = T\left[\frac{d}{dx}\left(\frac{dy}{dx}\right)\right]\delta x$$

$$R = T\frac{d^2y}{dx^2}\delta x \text{------}(2)$$

If m – mass per unit length of string,

Mass of element δx is= m δx

Acceleration in y direction= $\frac{d^2y}{dt^2}$

EQUATION OF MOTION OF A VIBRATING STRING:

$$By\ Newtons\ second\ law\ of\ motion$$

Force=mass x acceleration

$$= m\delta x.\frac{d^2y}{dt^2} \quad \text{------(3)}$$

From equation (2) and (3)

$$m\delta x.\frac{d^2y}{dt^2} = T\frac{d^2y}{dx^2}\delta x$$

$\frac{d^2y}{dt^2} = \frac{T}{m}\frac{d^2y}{dx^2}$ This is differential eqn of Vibrating string

SUMMERY:

Transverse vibration of string is chief source of musical sound.

Under displaced position resultant downward components of tension = $R = T\frac{d^2y}{dx^2}\delta x$ for length δx

Force acting on element δx according to Newton

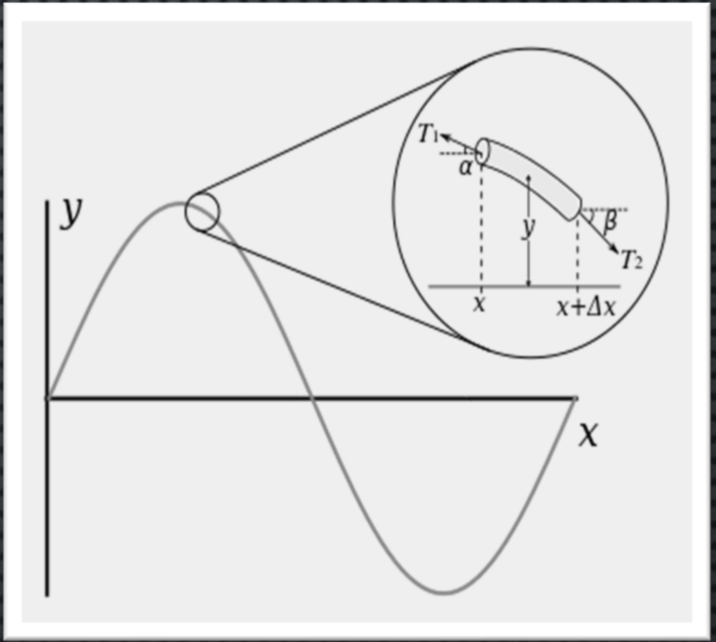
$$m\delta x.\frac{d^2y}{dt^2}$$

Under displaced equilibrium condition

$$m\delta x.\frac{d^2y}{dt^2} = T\frac{d^2y}{dx^2}\delta x$$

$$\frac{d^2y}{dt^2} = \frac{T}{m}\frac{d^2y}{dt^2}$$

WAY TOWARDS SOLUTION…….



Aim: To study motion of vibrating string
Un displaced position of string
Displaced position of string
Under displaced equilibrium condition of string Forces acting on Small element δx are

1)Resultant downward restoring force due to tension T in string.

2) Resultant upward force due to acceleration in string

Under displaced equilibrium condition of string equating Forces we get equation of motion of string

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WAVES

VELOCITY OF TRANSVERSE WAVES ALONG A STRETCHED STRING

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WHAT WE HAVE LEARN :

1. Differential equation of wave motion

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2}$$

2. Differential equation of stretched string

$$\frac{d^2y}{dt^2} = \frac{T}{m} \frac{d^2y}{dx^2}$$

VELOCITY OF TRANSVERSE WAVES ALONG A STRETCHED STRING:

Assumptions:

1. Length of string –greater than diameter

2 Perfectly uniform and flexible

3 Stretched between two fixed points

4 Tension in string should be large

5 Effect of gravitational force can be neglected.

6. Tension T in string should be constant everywhere

VELOCITY OF TRANSVERSE WAVES ALONG A STRETCHED STRING :

Portion PABQ of string in which transverse wave is traveling

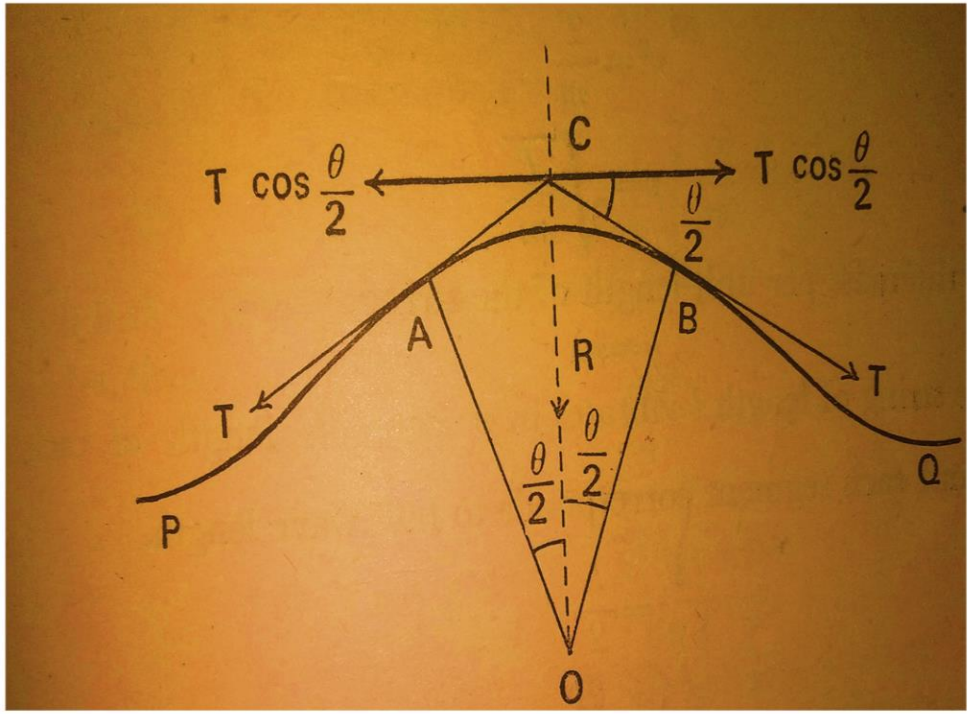
AB –Small element of string

O- Centre of curvature of AB

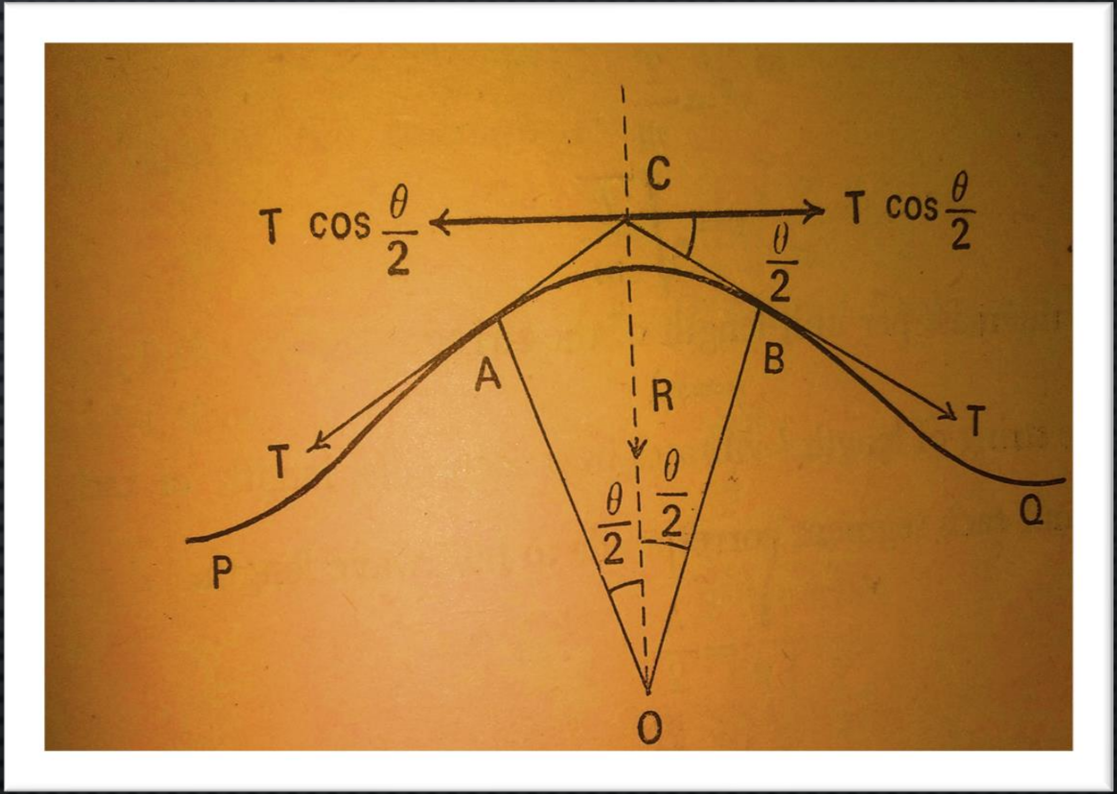
As curvature is small, θ small

T- Tension at A and B

tangential TO element at A and B



VELOCITY OF TRANSVERSE WAVES ALONG A STRETCHED STRING :



Resolving tensions at A

1) $T\,sin\frac{\theta}{2}$ Perpendicular to string

2) $T\,cos\frac{\theta}{2}$ parallel to string

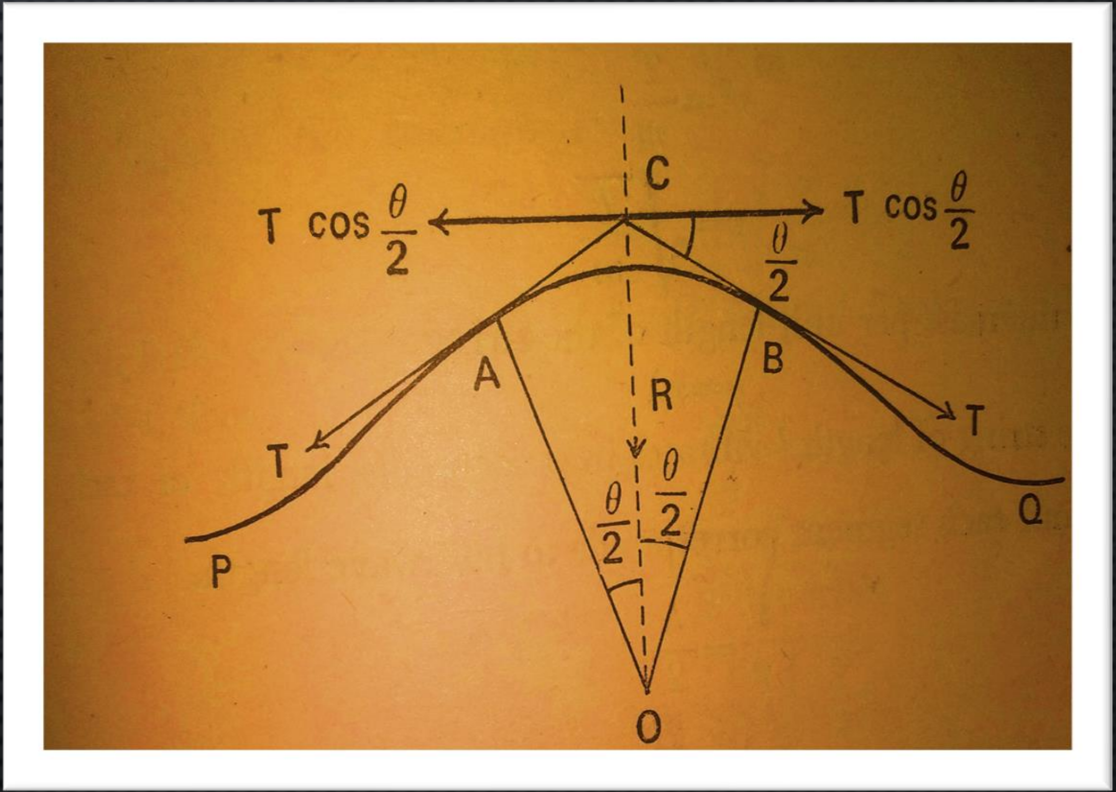
Similarly

Resolving tensions at B

1) $T\,sin\frac{\theta}{2}$ Perpendicular to string

2) $T\,cos\frac{\theta}{2}$ parallel to string

VELOCITY OF TRANSVERSE WAVES ALONG A STRETCHED STRING :



Parallel components cancels

Resultant perpendicular components along CO

Resultant tension along CO

$= 2T \sin\frac{\theta}{2}$

as θ small $\sin\frac{\theta}{2} = \frac{\theta}{2}$

VELOCITY OF TRANSVERSE WAVES ALONG A STRETCHED STRING :

Resultant tension= $2T\,\frac{\theta}{2}$ = $T\theta$-------(1)

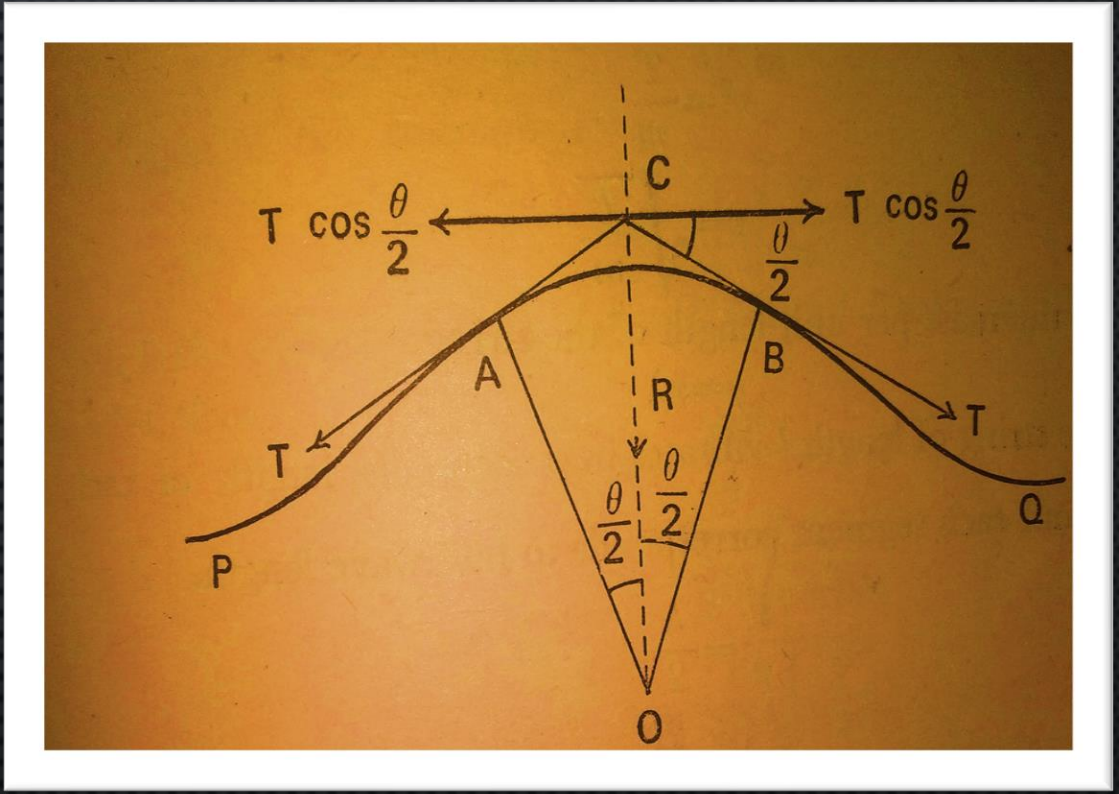
For equilibrium position

Resultant tension provides necessary centripetal force

$= \frac{(m\delta x)v^2}{R}$ -------(2)

$\frac{(m\delta x)v^2}{R}$ = $T\theta$

VELOCITY OF TRANSVERSE WAVES ALONG A STRETCHED STRING :



But from fig Sinθ=$\frac{\delta x}{R}$ = θ=$\frac{\delta x}{R}$

$$\frac{(m\delta x)v^2}{R} = T\frac{\delta x}{R}$$

$$v^2 = \frac{T}{m}$$

$$v = \sqrt{\frac{T}{m}} \text{----(3)}$$

Velocity of transverse waves along stretched string

FREQUENCY AND PERIOD OF VIBRATING STRING:

Let string fixed at both ends.

Plucked at Centre

Transverse waves set up in string

Reflected at boundary or fixed point and reverse

During one complete vibration wave travels twice length of string

$$n = \frac{v}{\lambda} = \frac{v}{2l} = \frac{1}{2l}\sqrt{\frac{\boldsymbol{T}}{\boldsymbol{m}}}$$

FREQUENCY AND PERIOD OF VIBRATING STRING:

n= $\frac{1}{2l}\sqrt{\frac{T}{m}}$ is the lowest mode of vibration called fundamental mode

Period of vibration of fundamental mode

$$T = \frac{1}{n} = 2l\sqrt{\frac{m}{T}}$$

If D be diameter of wire

ρ - density of material of wire

FREQUENCY AND PERIOD OF VIBRATING STRING:

Mass per unit length $m = \frac{Mass}{Length} = \frac{density \times Volume}{Length} = \frac{\rho \pi r^2 l}{l}$

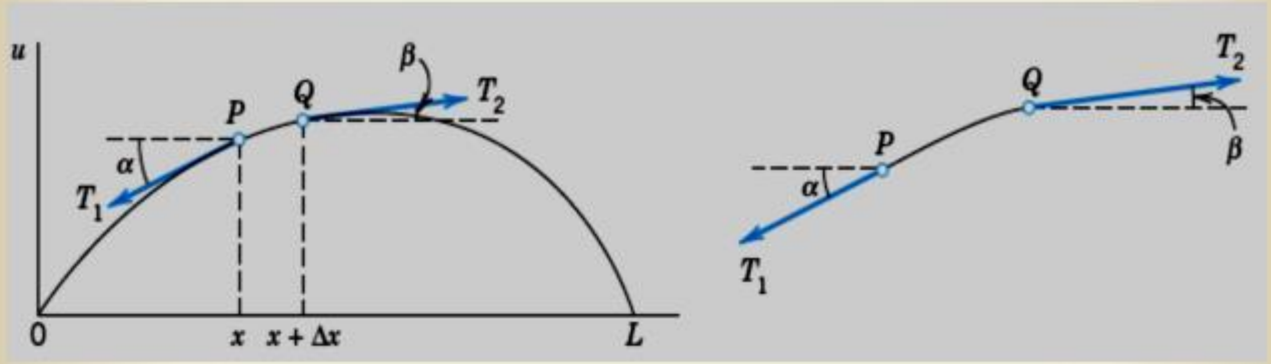
$$\frac{\rho \pi r^2 l}{l} = \frac{\rho \pi D^2}{4}$$

Frequency of fundamental mode $n = \frac{1}{2l}\sqrt{\frac{4T}{\rho \pi D^2}} = \frac{1}{Dl}\sqrt{\frac{T}{\rho \pi}}$

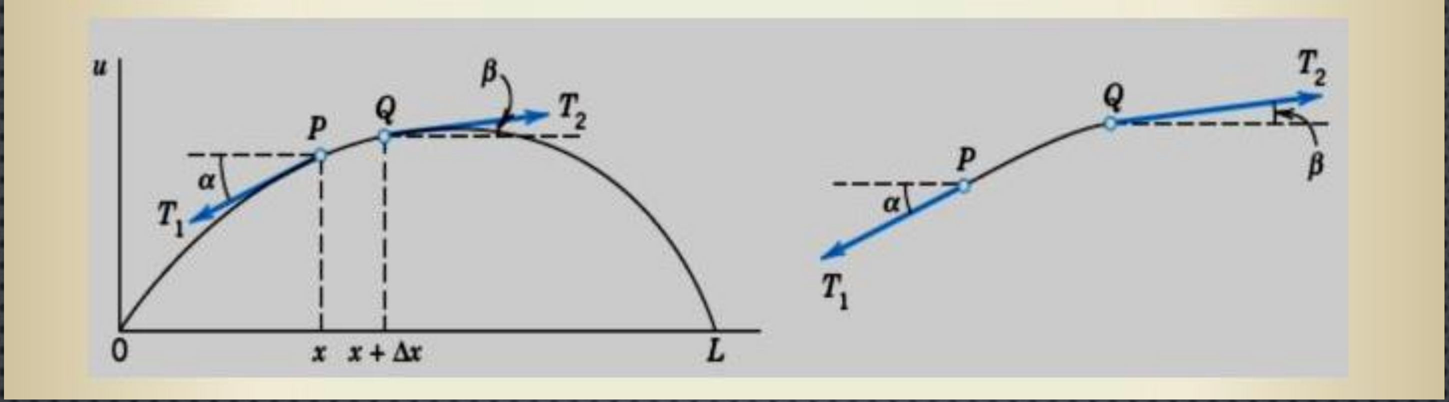
Period $T = \frac{1}{n} = Dl\sqrt{\frac{\rho \pi}{T}}$

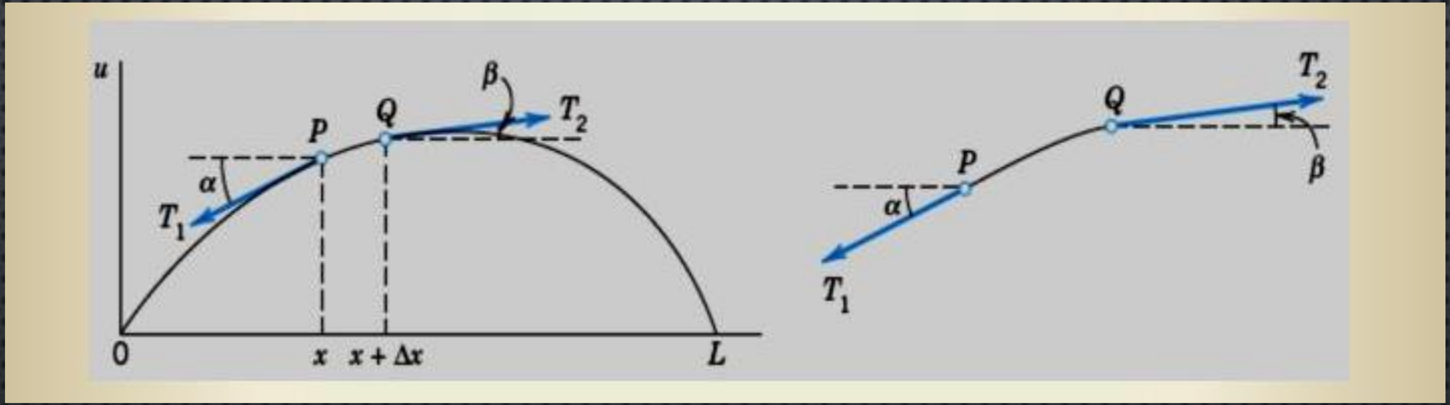
THANK YOU

Wave Equation
Modeling of Vibrating String



- The tension is tangential to the curve
- T_1 and T_2 are the tension at the endpoints P and Q
- ρ is the mass of the string per unit length
- No motion in the horizontal direction


u
P
Q
β
T_2
α
T_1
0
x
x + Δx
L
P
Q
T_2
β
α
T_1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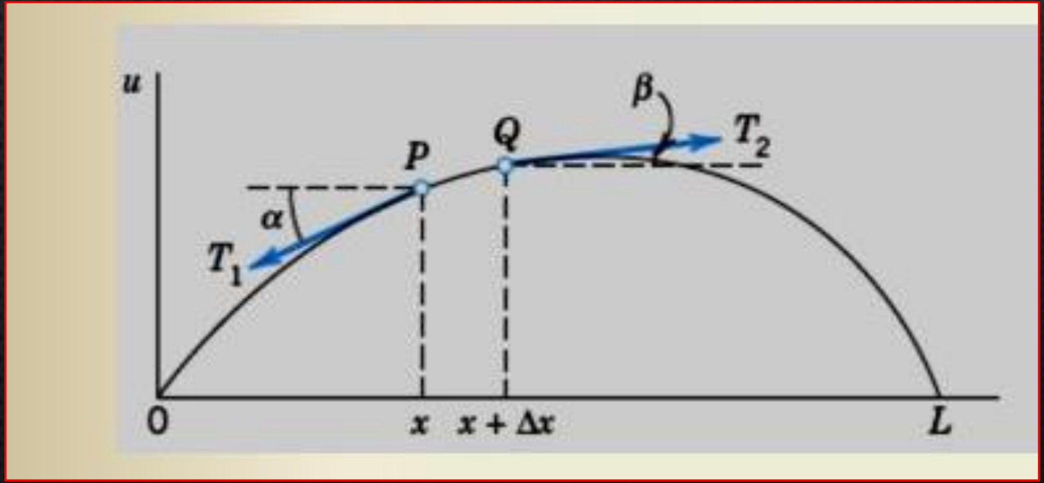
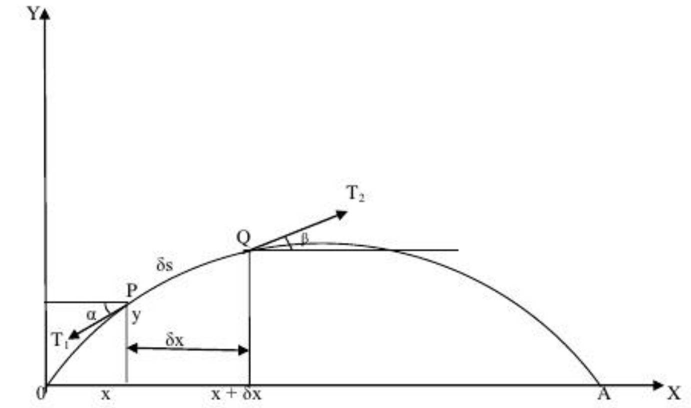
α
T_1


u
P
Q
β
T_2
α
T_1
0
x
x + Δx
L

One dimensional Wave Equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

(Vibrations of a stretched string)



Consider a uniform elastic string of length *l* stretched tightly between points O and A and displaced slightly from its equilibrium position OA. Taking the end O as the origin, OA as the axis and a perpendicular line through O as the y-axis, we shall find the displacement y as a function of the distance x and time t.

Assumptions

(i) Motion takes places in the XY plane and each particle of the string moves perpendicular to the equilibrium position OA of the string.

(ii) String is perfectly flexible and does not offer resistance to bending.

(iii) Tension in the string is so large that the forces due to weight of the string can be neglected.

(iv) Displacement y and the slope $\frac{\partial y}{\partial x}$ are small, so that their higher powers can be neglected.

ONE DIMENSIONAL WAVE EQUATION

Question 1: With necessary physical assumptions derive the one dimensional wave equation $u_{tt} = c^2 u_{xx}, 0 \le x \le L, t \ge 0.$

Consider a string of homogeneous material. Stretch the string to a length L and fix the string at its end points. Distort the string (initial displacement $u(x,0) = f(x), 0 \le x \le L$ and initial velocity $u_t(x,0) = g(x), 0 \le x \le L$)at time t=0 and allow it to vibrate. We shall make the following simplifying physical assumptions:

- Mass per unit length 'ρ 'of the string is constant(Offers no resistance for bending, perfectly elastic).
- The applied tension to stretch the string before fixing it at its ends is so large that the effect of gravitational force can be neglected.
- String particles undergo vibrations only in a vertical plane(Displacement remains small in magnitude).

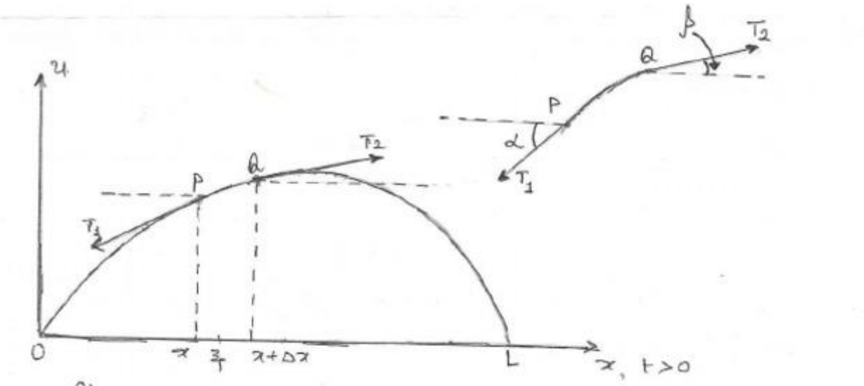


figure 1. Deflected string at fixed time t > 0.

Consider the forces acting on a small portion PQ of length $\triangle x$ at any time $t > 0$. Let T_1 and T_2 denotes the tension at the end points P and Q of that portion. Then $-T_1 sin\alpha$ and $T_2 sin\beta$ are the vertical components of T_1 and T_2 respectively.(See figure 1).

Since there is no motion in the horizontal plane, the horizontal components of T_1 and T_2 are constant. Therefore

(1) $$T_1 cos\alpha = T_2 cos\beta = T = const.$$

By Newtons second law of motion, Force=mass$\star$acceleration.

Thus,

$$T_2 sin\beta - T_1 sin\alpha = \rho(\triangle x) u_{tt}(\xi, t) \text{ ,for some } \xi, x \le \xi \le x + \triangle x$$

i.e. $tan\beta - tan\alpha = \frac{\rho \triangle x}{T} u_{tt}(\xi, t)$, (using (1) and dividing by T).

Taking limit as $\triangle x \longmapsto 0$,

we get $u_{xx} = \frac{\rho}{T} u_{tt}$

3.2 Wave and Heat equations

We derive one dimentional wave equation which is due to the transverse vibration of a stretched string. We also derive one dimensional heat equation which is due to the heat flow along a thin bar insulated on all sides. We also discuss the solution of these two equations.

3.21 Derivation of one dimensional wave equation

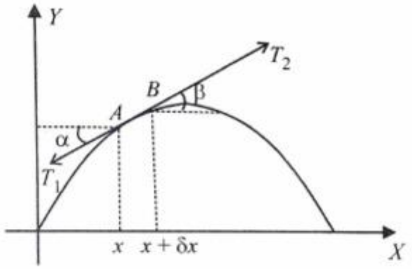
Consider a flexible string tightly stretched between two fixed points at a distance l apart. Let ρ be the mass per unit length of the string.

We shall assume the following.

(i) The tension T of the string is same throughout.

(ii) The effect of gravity can be ignored due to large tension T.

(iii) The motion of the string is in small transverse vibrations.



Let us consider the forces acting on a small element AB of length δx.

Let T_1 and T_2 be the tensions at the points A and B.

Since there is no motion in the horizontal direction, the horizontal components T_1 and T_2 must cancel each other.

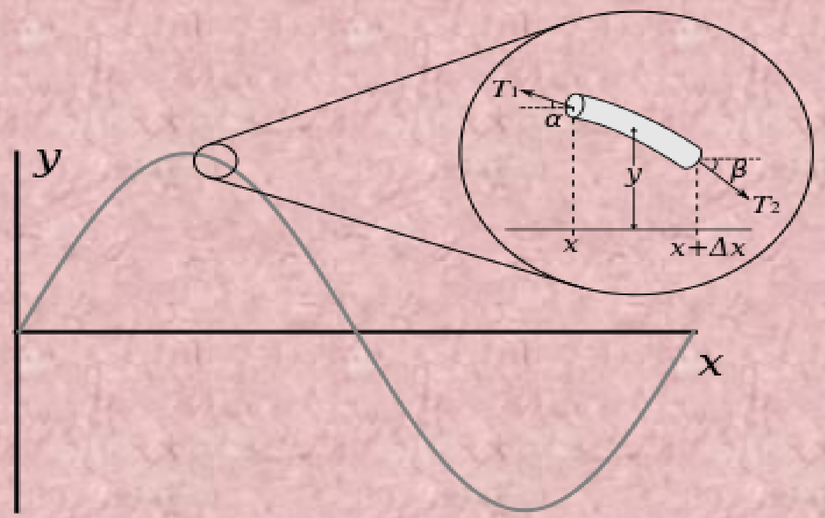
$$\therefore \quad T_1 \cos\alpha = T_2 \cos\beta = T \qquad \ldots (i)$$

where α and β are the angles made by T_1 and T_2 with the horizontal. Vertical components of tension are $-T_1 \sin\alpha$ and $T_2 \sin\beta$, where the negative sign is used because T_1 is directed downwards. Hence the resulant force acting vertically upwards is $T_2 \sin\beta - T_1 \sin\alpha$.

Applying Newton's second law of motion, that is

Force = mass $\times$ acceleration, we get

$$T_2 \sin\beta - T_1 \sin\alpha = (\rho\,\delta x)\,\frac{\partial^2 u}{\partial t^2}$$



y
x
T1
α
y
β
T2
x
x+Δx