UNIT IV

RELATIVITY

BY

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INTRODUCTION:

Albert Einstein showed that measurement of time and space are affected by motion between observer and object

Relativity connects

space and time

Matter and energy

Electricity and magnetism

FRAME OF REFERENCE:

Frame of reference: A system of co-ordinate axes which defines position of particle in two or three dimensional space

- Part of description of motion
- Something is moving always implies specific frame of reference
- An inertial frame of reference in which Newtons first law of motion holds
- Any frame of reference that moves at constant velocity relative to inertial frame is itself an inertial frame

FRAME OF REFERENCE:

All inertial frames are equally valid

All constant velocity motion is relative

There is no universal frame of reference that can be used everywhere

Special theory of relativity (1905) –treat problems that involve inertial frames of reference

General theory of relativity(1915)- treat problems that involve non-inertial frames of reference or accelerated frames of reference

POSTULATES OF SPECIAL RELATIVITY:

Two postulates

1) Principle of relativity : Laws of physics are the same in all frames of reference

Follows from absence of universal frame of reference

If laws of motion were different for different observers in relative motion then observer could not distinction which of them is stationary and which is motion.

Principle of relativity expresses this fact

POSTULATES OF SPECIAL RELATIVITY:

2) Speed of light postulate: The speed of light in free space has the same value in all inertial frames of reference.

This postulate is based on results of many experiments

Speed of light is 2.998×10^8 m/s

This postulate differentiate classical and quantum theory

GALILEAN TRANSFORMATION:

Suppose we are in inertial frame of reference S

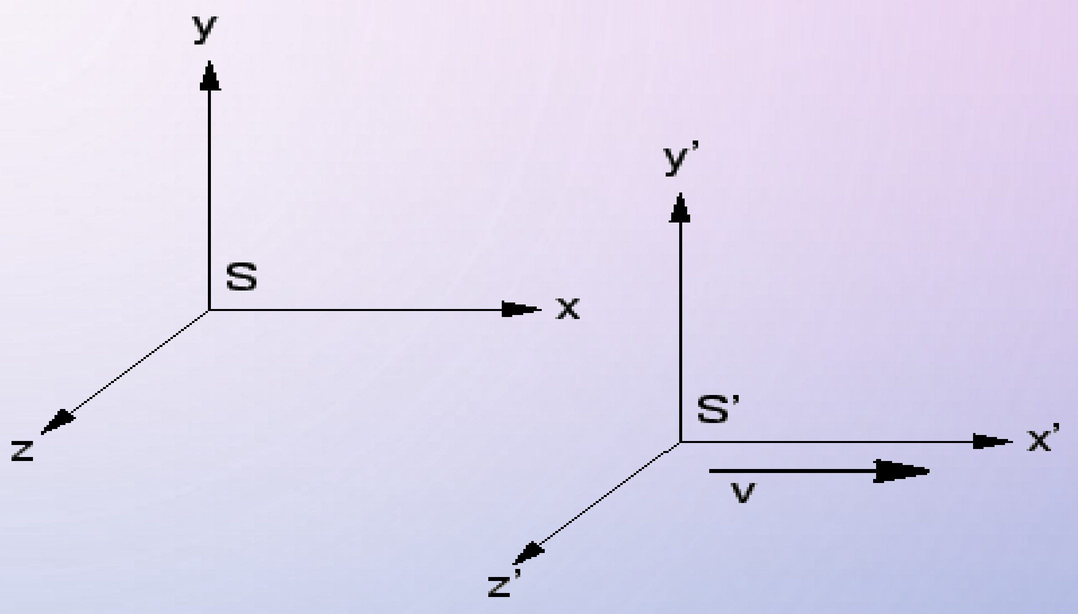
Coordinates of event at t are *x* , *y*, *z*

An observer located in different inertial frame S′ moving with respect to S at constant velocity *v*

Coordinates of same event at time *t*′ are *x*′ ,*y*′ , *z*′

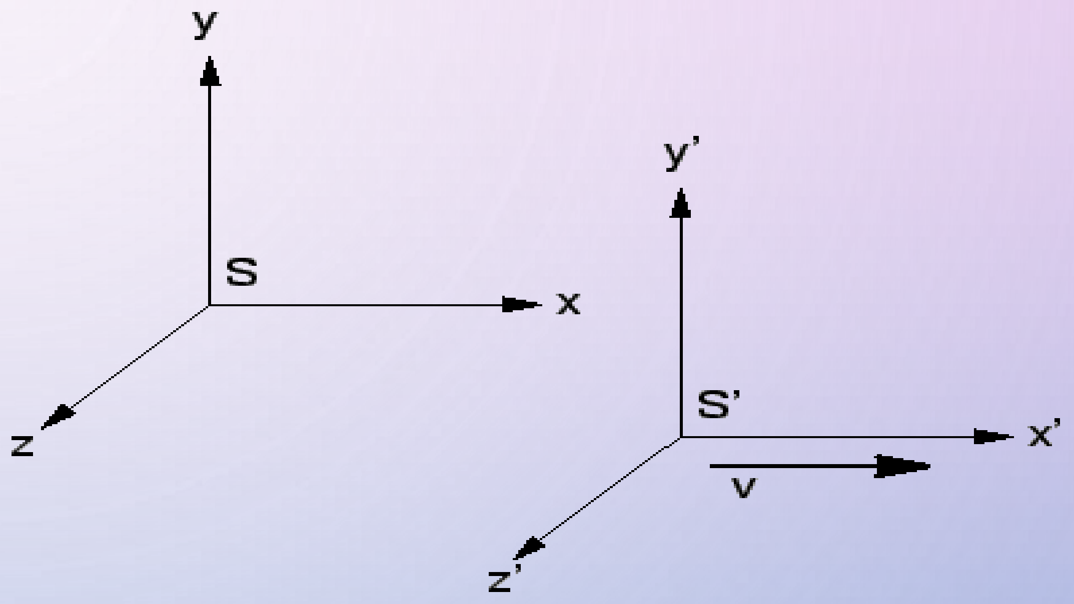
For simplicity v is in +*x* direction

How *x*, *y*, *z*, *t* are related to *x*′ ,*y*′ , *z*′ ,t ′



GALILEAN TRANSFORMATION:

Before special relativity, transforming measurement from one inertial frame to other seemed obvious.



If clocks in both systems are started when origins of S and S′ coincides, measurements in the x direction made in S will be greater than those in S′ by an amount vt ,which is the distance S′ has moved in the x direction

$$x' = x - vt \qquad \text{---(1)}$$

GALILEAN TRANSFORMATION:

There is no relative emotion in y and z direction

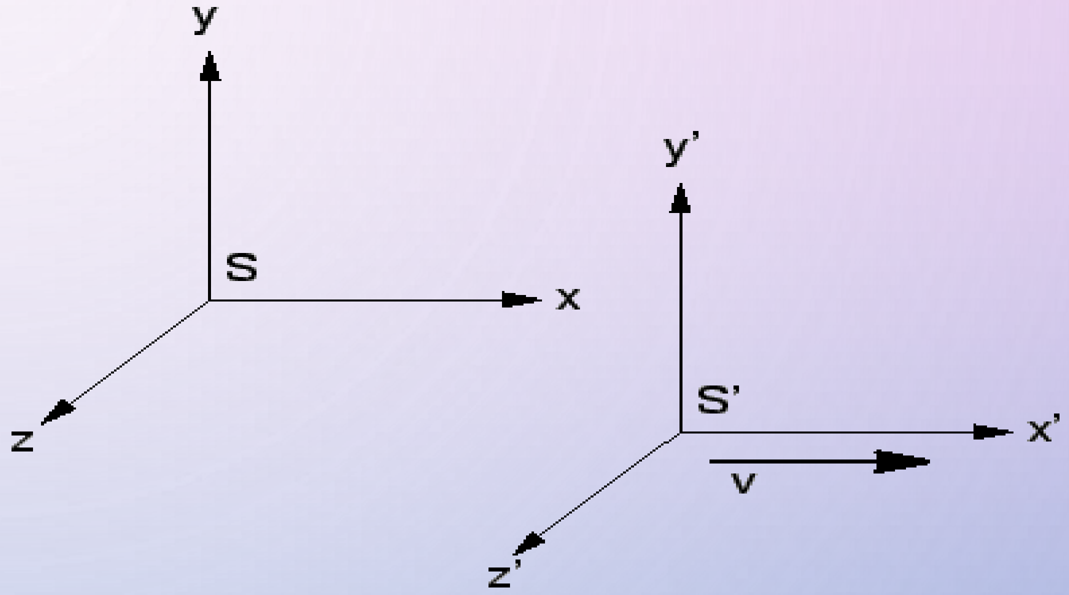
So $y' = y$ --(2)

And $z' = z$ ---(3)

With our everyday experience

$t' = t$ ----(4)

Equation (1), (2), (3) and (4) are Galilean transformation



GALILEAN TRANSFORMATION:

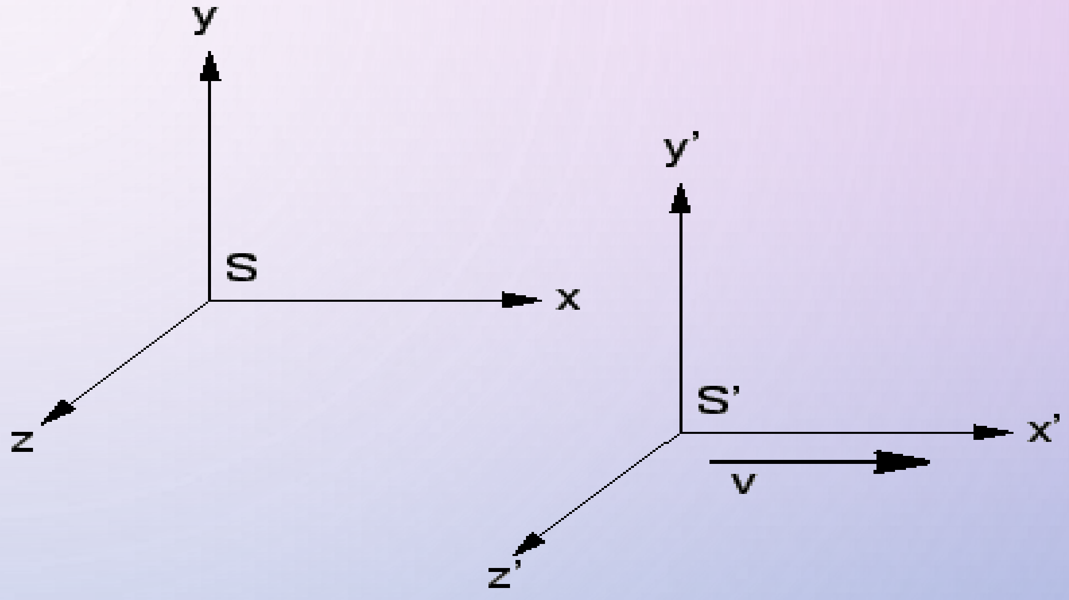
According to Galilean transformation Velocity components in S frame and to their equivalent in s' frame are

$$v_x' = \frac{dx'}{dt'} = v_x - v \text{ ----(5)}$$

$$v_y' = \frac{dy'}{dt'} = v_y \text{ -----(6)}$$

$$v_z' = \frac{dz'}{dt'} = v_z \text{ ---------(7)}$$

Although it looks straightforward but violate postulates of special relativity



LORENTZ TRANSFORMATION:

Nature of correct relationship between x and x' is

$x' = k(x - vt)$ ----(1)

k does not depends upon either $x\ or\ t$ but may be function of v

Because of equations of physics are same in both S and S$'$

Corresponding equation for x in terms x' and t'

$x = k(x' + vt')$ $-$---(2)

k must be same for both S and S$'$ except direction of v

Perpendicular to direction of v there is no difference in

$y\ and\ y'$ and $z\ and\ z'$

LORENTZ TRANSFORMATION:

$y = y'$ -----(3)

$z = z'$ ------(4)

Time coordinates t and t' are not the same

Substituting values of x' in eq.(2)

$x = k^2(x - vt) + kvt'$

From which we find

$t' = kt + \left(\frac{1-k^2}{kv}\right)$x----(5)

Eq. 1 to 5 constitutes a coordinate transformation

Satisfies first postulate of special relativity

LORENTZ TRANSFORMATION:

Using second postulate

At $t = 0$, origins of both S and S$'$ coincides

Flare is set off at common origin S and S$'$ at $t = 0$,

Observers in both measures same speed of flares of light spread out

Both must find same speed c

In frame S

$x = ct$ -----(6)

In frame S$'$

$x' = ct'$ ----(7)

LORENTZ TRANSFORMATION:

Substituting x' and t' in eq. (7) with eq. (1) and (5)

$$k(x - vt) = ckt + \left(\frac{1 - k^2}{kv}\right)cx$$

Solving for x

$$x = \frac{ckt + vkt}{k - \left(\frac{1 - k^2}{kv}\right)c}$$

LORENTZ TRANSFORMATION:

$$x = \frac{ckt + vkt}{k - \left(\frac{1 - k^2}{kv}\right)c}$$

$$= ct\left[\frac{k + \frac{v}{c}k}{k - \left(\frac{1 - k^2}{kv}\right)c}\right]$$

LORENTZ TRANSFORMATION:

$$= ct\left[\frac{k+\frac{v}{c}k}{k-\left(\frac{1-k^2}{kv}\right)c}\right]$$

$$x = ct\left[\frac{1+\frac{v}{c}}{1-\left(\frac{1}{k^2}-1\right)\frac{c}{v}}\right] - -(8)$$

LORENTZ TRANSFORMATION:

Eq. (8) is same as eq.(6) $x = ct$ provided bracket term equal to 1

$$\left[\frac{1+\frac{v}{c}}{1-\left(\frac{1}{k^2}-1\right)\frac{c}{v}}\right] = 1$$

And

$$k = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} --(9)$$

LORENTZ TRANSFORMATION:

Putting the value of k

Complete transformation measurement of an event made in S corresponding measurement made in S'

Lorentz Transformation

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} --(10)$$

$$y = y' \quad \text{-----}(11)$$

$$z = z' \quad \text{-----}(12)$$

LORENTZ TRANSFORMATION:

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad --(13)$$

Lorentz transformation reduces to Galilean transformation when relative velocity v is very less than velocity of light.

TIME DILATION:

A MOVING CLOCK TICKS MORE SLOWLY THAN CLOCK AT REST

Relative motion affect measurement of time interval

Clock that moves wrt observer ticks more slowly than rest

All processes occurs more slowly to an observer in different inertial frame

t_0 is the time interval between two events in spacecraft

t is same interval has longer time on ground

Proper Time : Event that occurs at same place in observers frame of reference is called proper time t_0

From ground same event appears longer than proper time called time dilation

TIME DILATION:

A MOVING CLOCK TICKS MORE SLOWLY THAN CLOCK AT REST

Consider same two clocks

In each clock pulse a light pulse is reflected back and forth between two mirrors L_0 apart

Whenever light strikes the lower mirror, an electric signal is produced that mark recording tape

Each mark corresponds to one tick of ordinary clock

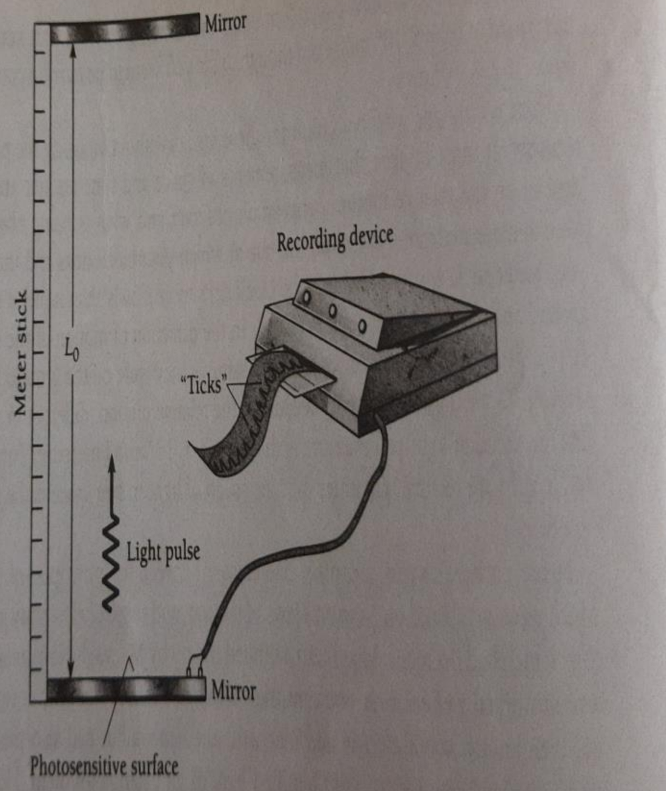


Figure 1.3 A simple clock. Each "tick" corresponds to a round trip of the light pulse from the lower mirror to the upper one and back.

TIME DILATION:

A MOVING CLOCK TICKS MORE SLOWLY THAN CLOCK AT REST

One clock is at rest in a laboratory on the ground

Other in space craft that moves at the speed v relative to ground

Observer in laboratory watches both clocks

Does find ticks at same rate?

TIME DILATION:

A MOVING CLOCK TICKS MORE SLOWLY THAN CLOCK AT REST

Laboratory clock in operation

Time interval between sticks is proper time t_0

Time required for light pulse to travel between mirrors= $t_0\ /2$

In time t_0 , distance travelled $= L_0$

Hence $\frac{t_0}{2} = \frac{L_0}{c}$ $\qquad c$ is velocity of light

$t_0 = \frac{2L_0}{c}$ --(1)

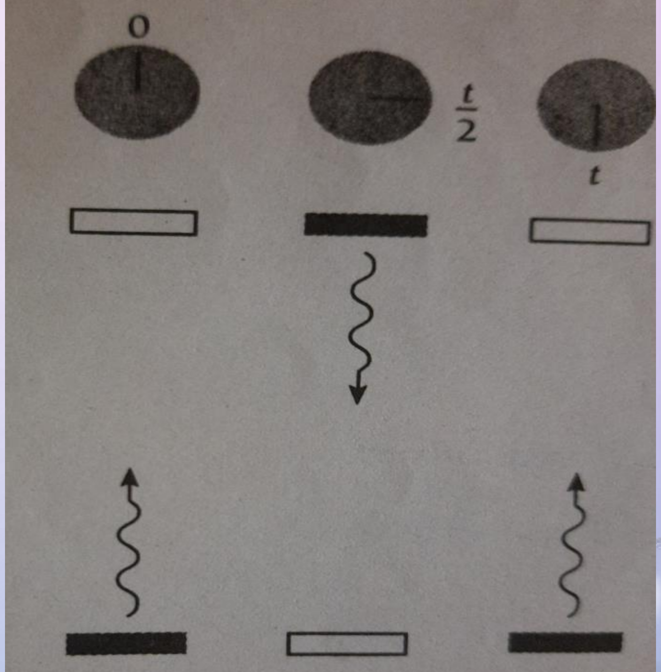


Figure 1.4 A light-pulse clock at rest on the ground as seen by an observer on the ground. The dial represents a conventional clock on the ground.

TIME DILATION:

A MOVING CLOCK TICKS MORE SLOWLY THAN CLOCK AT REST

Moving clock in operation

Moving clock with mirrors perpendicular to the direction of motion relative to ground

Time interval between ticks is t

Because of clock is moving , light pulse , as seen from ground, follows a zigzag path

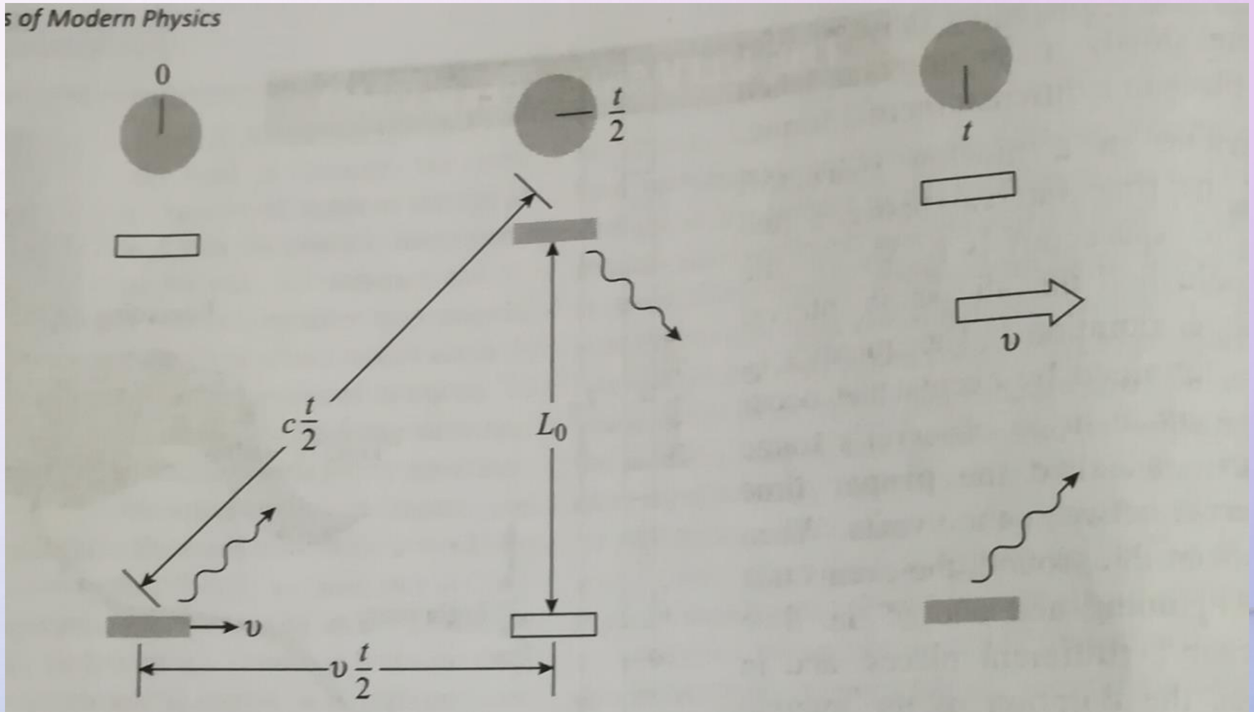


Fig 1.5 *A light-pulse clock in a spacecraft as men by an observer on the ground. The mirrors are parallel to the direction of motion of the spacecraft. The dial represents a conventional clock on the ground.*

TIME DILATION:

A MOVING CLOCK TICKS MORE SLOWLY THAN CLOCK AT REST

In $\frac{t}{2}$ time(from lower mirror to upper)

Horizontal distance travelled by light pulse $=\frac{vt}{2}$

Total distance= $\frac{ct}{2}$

L_0 is vertical distance between mirrors

$$\left(\frac{ct}{2}\right)^2 = L_0^2 + \left(\frac{vt}{2}\right)^2$$

$$\frac{t^2}{4}(c^2 - v^2) = L_0^2$$

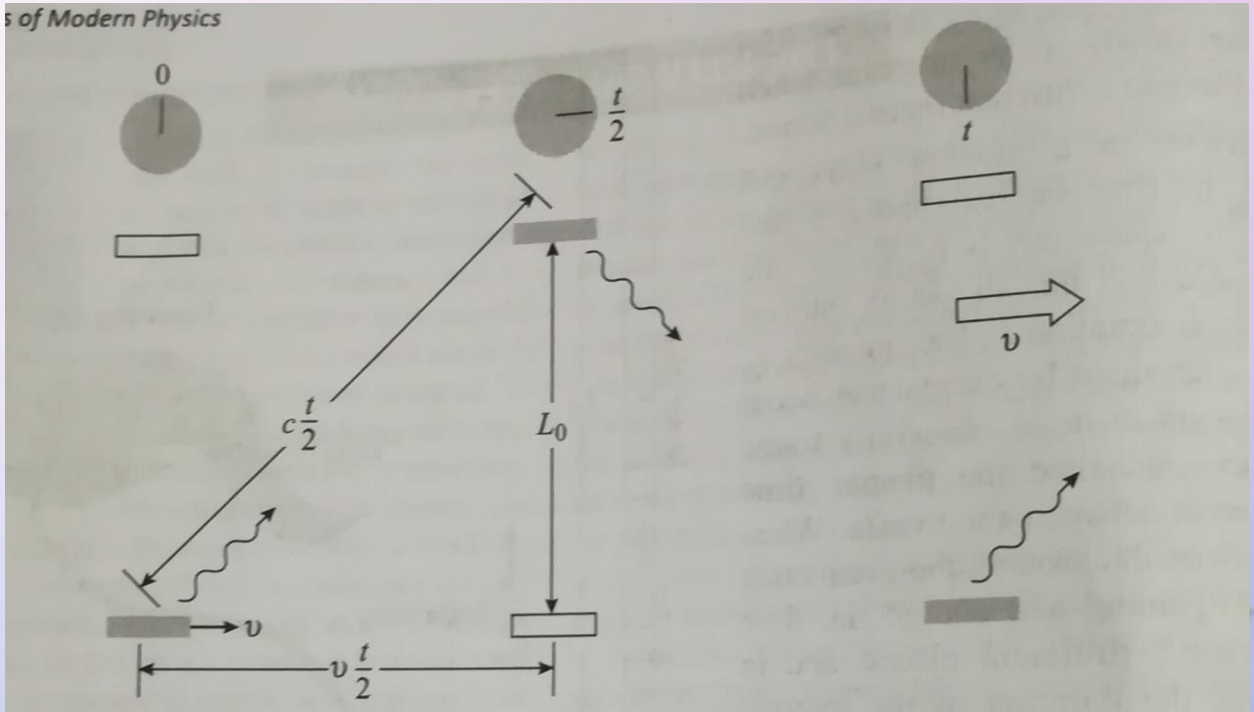


Fig 1.5 *A light-pulse clock in a spacecraft as men by an observer on the ground. The mirrors are parallel to the direction of motion of the spacecraft. The dial represents a conventional clock on the ground.*

TIME DILATION:

A MOVING CLOCK TICKS MORE SLOWLY THAN CLOCK AT REST

$$t^2 = \frac{4L_0^2}{(c^2 - v^2)}$$

$$= \frac{(2L_0)^2}{c^2\left(1 - \frac{v^2}{c^2}\right)}$$

$$t = \frac{\frac{2L_0}{c}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

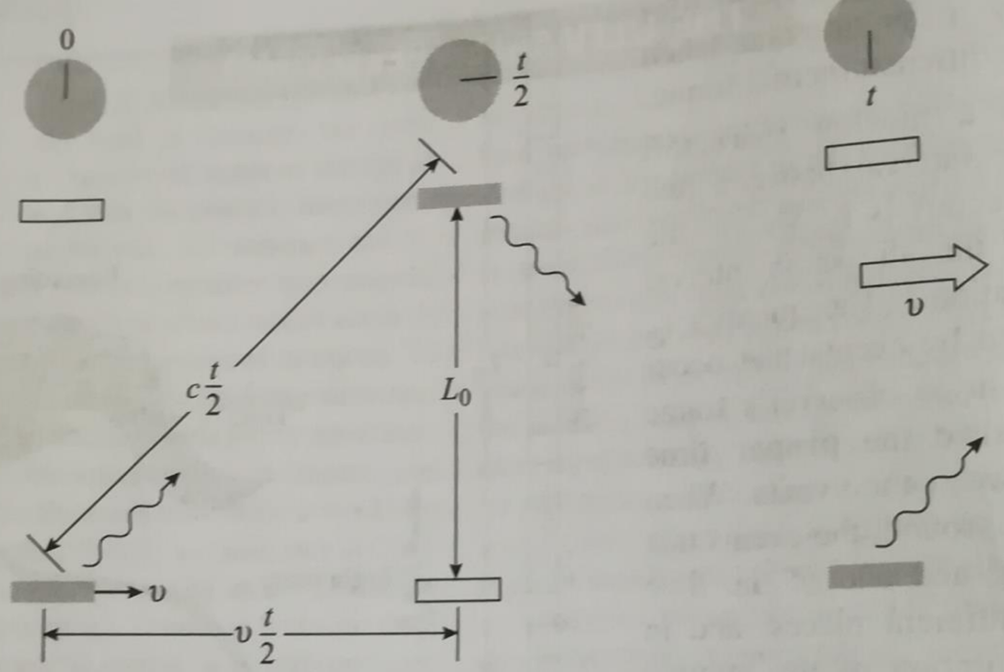


Fig 1.5 *A light-pulse clock in a spacecraft as men by an observer on the ground. The mirrors are parallel to the direction of motion of the spacecraft. The dial represents a conventional clock on the ground.*

TIME DILATION:

A MOVING CLOCK TICKS MORE SLOWLY THAN CLOCK AT REST

$\frac{2L_0}{c}$ is time interval t_0 between ticks on clock on ground

Time dilation:

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}} --(2)$$

As $\sqrt{1-\frac{v^2}{c^2}}$ is always less than 1

t is greater than t_0

Moving clock in a space craft appears to tick at a slower rate than stationary on the ground , as seen by observer on ground

LENGTH CONTRACTION:

FASTER MEANS SHORTER

Measurement of length as well as time intervals are affected by relative motion

Length L of object in motion wrt observer at rest appears shorter than its length L_0

Contraction occurs only in the direction of relative motion

Length L_0 in its rest frame called **Proper length**

Length contraction:

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

LENGTH CONTRACTION:

FASTER MEANS SHORTER

Like time dilation, the length contraction is reciprocal effect

Proper length L_0 found in the rest frame is the maximum length any observer will measure

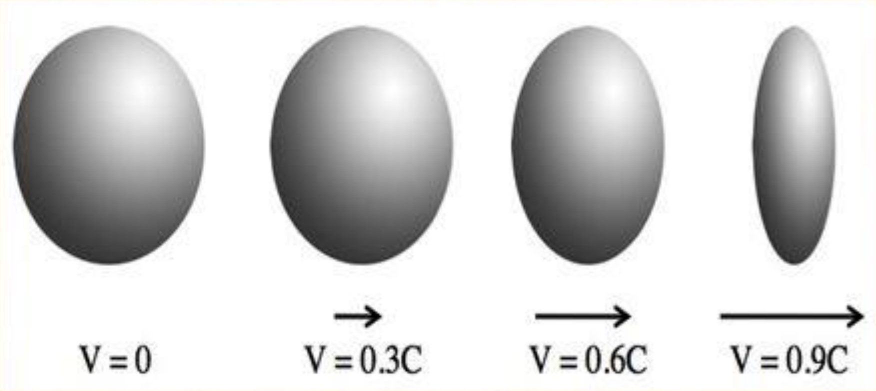
Length Contraction

- According to the theory of Special Relativity, objects appear to contract in the direction they are traveling when they are moving as fast as the speed of light.

$$L = L_o \sqrt{1 - \frac{v^2}{c^2}}$$

L = length of object moving at speed v

L_o = length of object at rest



VELOCITY ADDITION :

Suppose something is moving relative to both S and S'

Observer in S measures its three velocity components

$$v_x = \frac{dx}{dt} \qquad v_y = \frac{dy}{dt} \qquad v_z = \frac{dz}{dt}$$

Observer in S' measures

$$v'_x = \frac{dx'}{dt'} \qquad v'_y = \frac{dy'}{dt'} \qquad v'_z = \frac{dz'}{dt'}$$

VELOCITY ADDITION :

Differentiating inverse Lorentz Transformation equations for $x, y, z \; and \; t$

$$dx = \frac{dx' + v dt'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$dy = dy'$$

$$dz = dz'$$

$$dt = \frac{dt' + \frac{v dx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

VELOCITY ADDITION :

$$v_x = \frac{dx}{dt} = \frac{dx' + v dt'}{dt' + \frac{v dx'}{c^2}}$$

$$= \frac{\frac{dx'}{dt'} + v}{1 + \frac{v}{c^2}\frac{dx'}{dt'}}$$

Relativistic velocity transformation

$$v_x = \frac{v'_x + v}{1 + \frac{v v'_x}{c^2}} \quad ---(1)$$

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VELOCITY ADDITION :

$$v_y = \frac{v_y'\sqrt{1-\frac{v^2}{c^2}}}{1+\frac{vv_x'}{c^2}} \text{ ----(2)}$$

$$v_y = \frac{v_y'\sqrt{1-\frac{v^2}{c^2}}}{1+\frac{vv_x'}{c^2}} \text{----(3)}$$

RELATIVITY OF MASS :

REST MASS IS LEAST

When objects speed increases its mass also increases

$$Relativistic\ mass = m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Where m is mass of body moving at speed v

m_0 is rest mass of object

Mass of body moving at a speed of v relative to observer is larger than its mass when at rest relative to observer by factor $\sqrt{1 - v^2/c^2}$

Increase in mass is reciprocal

Relativistic mass increases are significant only at speeds nearly light

MASS- ENERGY RELATION:

E=MC[2]

Famous relationship obtained from postulates of special theory of relativity

Work done W on object by a constant force F acts through distance s is

$$W = F\,s$$

When object start from rest, all work done on it becomes K.E.

KE= $F\,s$

For variable force , KE is

$$KE = \int_0^s F\,ds$$

In nonrelativistic physics

KE of rest mass m_0 and speed v is $KE = \frac{1}{2}mv^2$

MASS- ENERGY RELATION:

E=MC²

To find correct relativistic formula for KE

Using Newtons second law of motion

$$KE = \int_0^s F\,ds = \int_0^s \frac{d(mv)}{dt}\,ds = \int_0^{mv} v\,d(mv) = \int_0^v v\,d\left(\frac{m_0 v}{\sqrt{1 - {v^2}/{c^2}}}\right)$$

Integrating by parts $\left(\int x\,dy = xy - \int y\,dx\right)$

$$KE = \frac{m_0 v^2}{\sqrt{1 - {v^2}/{c^2}}} - m_0 \int_0^v \frac{v\,dv}{\sqrt{1 - {v^2}/{c^2}}}$$

MASS- ENERGY RELATION:

E=MC²

$$KE = \frac{m_0 v^2}{\sqrt{1 - v^2/c^2}} - m_0 \int_0^v \frac{v\,dv}{\sqrt{1 - v^2/c^2}}$$

$$KE = \frac{m_0 v^2}{\sqrt{1 - v^2/c^2}} + \left[m_0 c^2 \sqrt{1 - v^2/c^2} \right]_0^v$$

$$KE = \frac{m_0 v^2}{\sqrt{1 - v^2/c^2}} - m_0 c^2$$

$$E = mc^2 - m_0 c^2$$

MASS- ENERGY RELATION:

E=MC²

$E = mc^2 - m_0c^2$ ---(1)

Result of above eq. states that KE of object is equal to increase in its mass due to relative motion multiplied by square of speed of light

Total energy

$mc^2 = m_0c^2 + KE$ ---(2)

MASS- ENERGY RELATION:

E=MC[2]

If we interpret mc^2 as total energy E of object, at rest KE=0 and it possesses the energy m_0c^2

$$E = E_0 + KE$$

Where

Rest energy $E_0 = m_0c^2$------(3)

If object is moving its total energy

$$E = mc^2 = \frac{m_0c^2}{\sqrt{1 - v^2/c^2}}$$