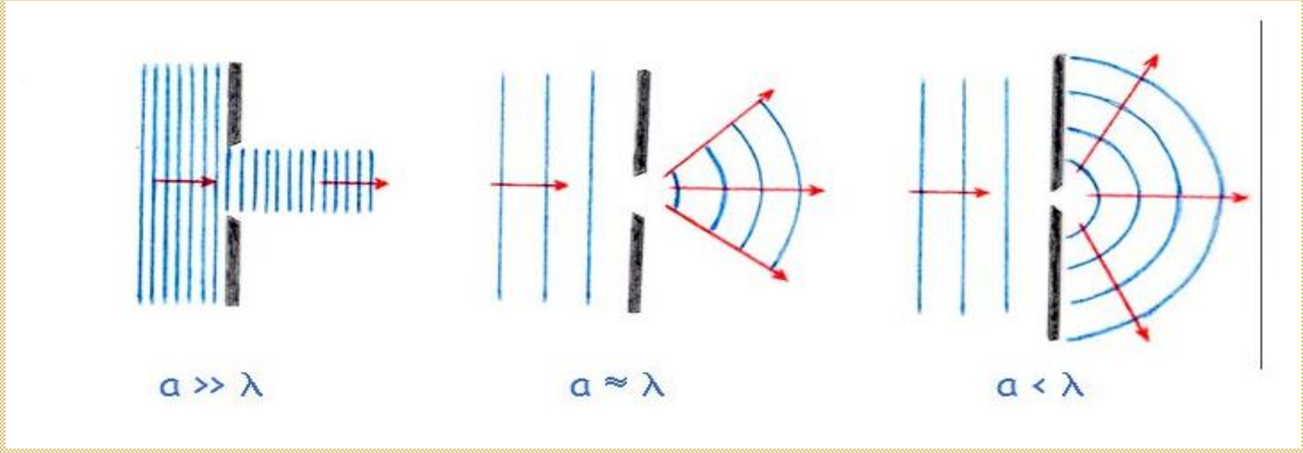
DIFFRACTION



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DIFFRACTION:

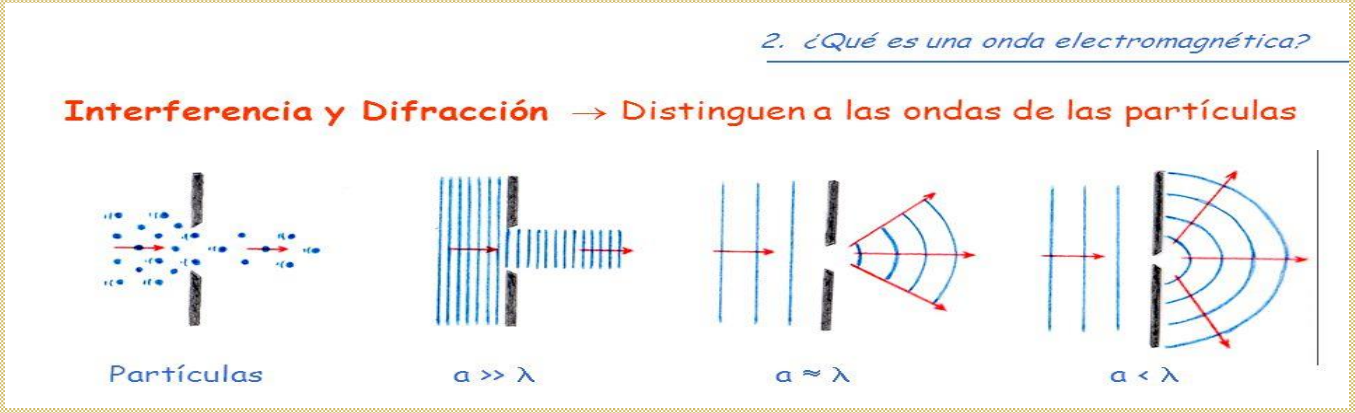
Diffraction: Bending of waves around the edges of an obstacle.

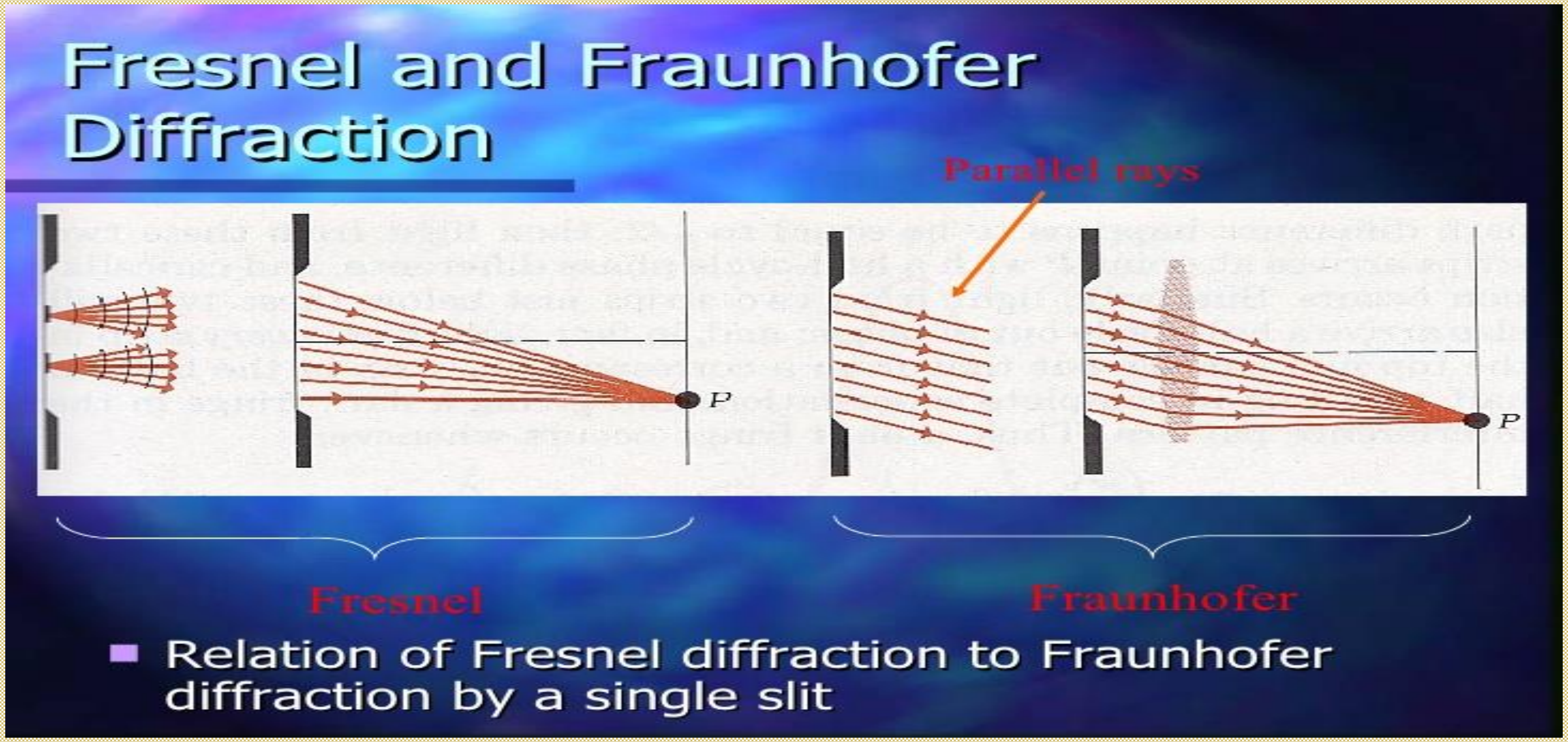
Diffraction is noticeable when the size of the obstacle or opening is comparable to a wavelength.

If the opening is large compared to wavelength, no bending

If the opening is small compared to wavelength ,b ending round the edges

If the opening is very small compared to wavelength, waves spread over the entire surface behind the opening





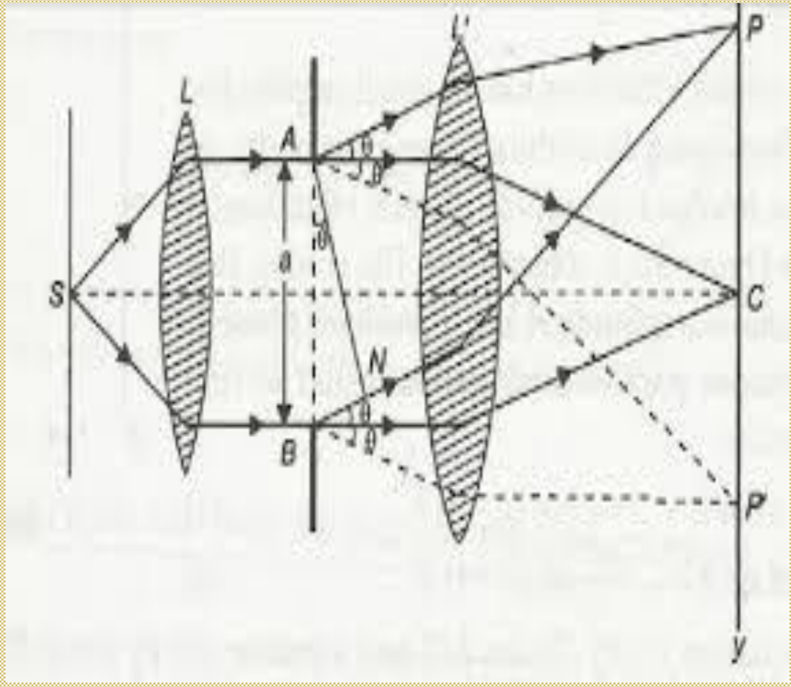
FRESNEL DIFFRACTION.

- Source and screen are at finite distances
- Observation does not required lens
- The incidence wave front is not planer

FRAUNHOFFER DIFFRACTION

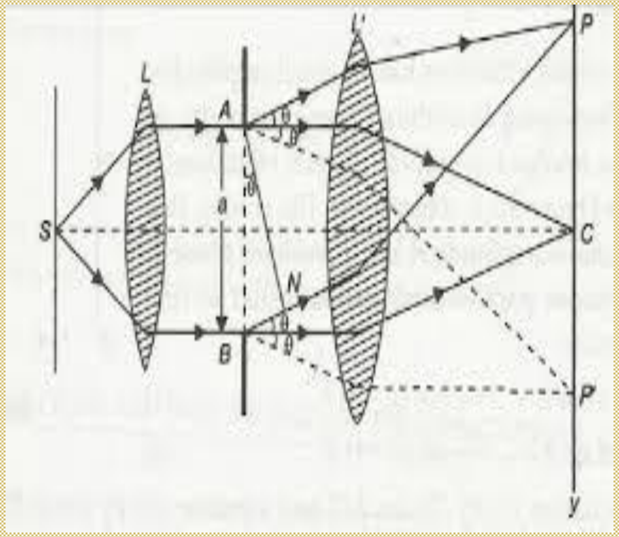
- Source and screen are at infinite distances
- Observation required lens
- Incident wave front is a plane

FRAUNHOFFER DIFFRACTION AT A SINGLE SLIT:



- Diagram:
- S   is narrow slit
- L1   is collimating lens
- AB   is slit of width "a"
- XY   is screen
- SC   is perpendicular to screen
- Working:
- Plane wave front incident on slit AB
- Secondary waves travelling parallel to SC come to focus at C
- Path difference between upper and lower halves is zero
- Point C is of point of maximum intensity

FRAUNHOFFER DIFFRACTION AT A SINGLE SLIT:



- Secondary waves travelling
  In a direction inclined at an angle
  θ to SC reach point P
- Point P will be max or min intensity
  depends on PD.
  PD= a sin θ
  If PD = λ,P will be of min. intensity
  The PD bet. Secondary waves from A and B is λ,PD bet. Secondary waves from Upper &Lower halves of AB is $\lambda/2$

$$a \sin \Theta_n = n\lambda$$

$$\sin \Theta_n = \frac{n\lambda}{a}$$

θn gives direction of nth minimum

Condition for maxima

$$sin\Theta n = \frac{(2n+1)\,{}^{\lambda}/_{2}}{a}$$

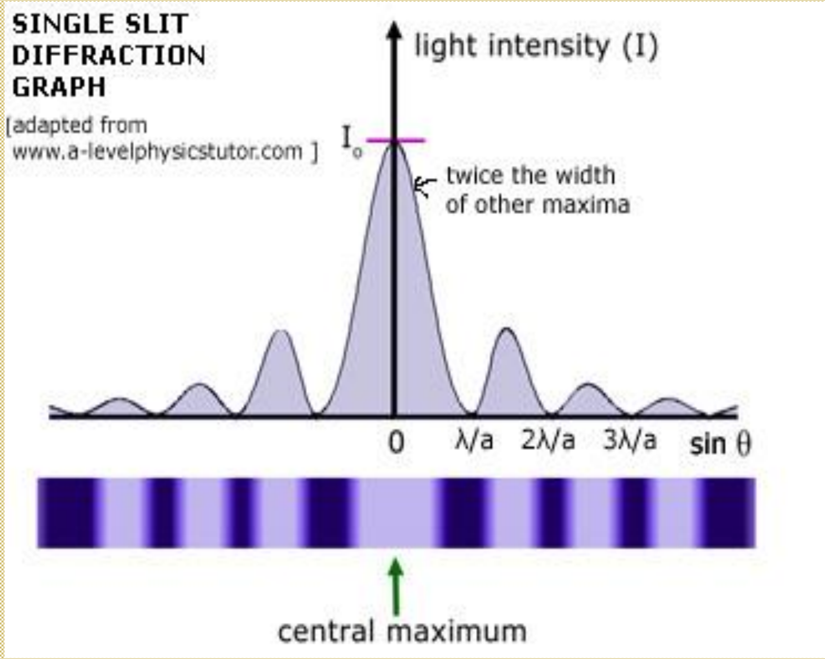
Fraunhoffer Diffraction pattern:

- Diffraction pattern due to single slit consist of a central bright Maxima at C followed by secondary maxima and minima on both sides

  Width of central maxima is proportional to wavelength of light.

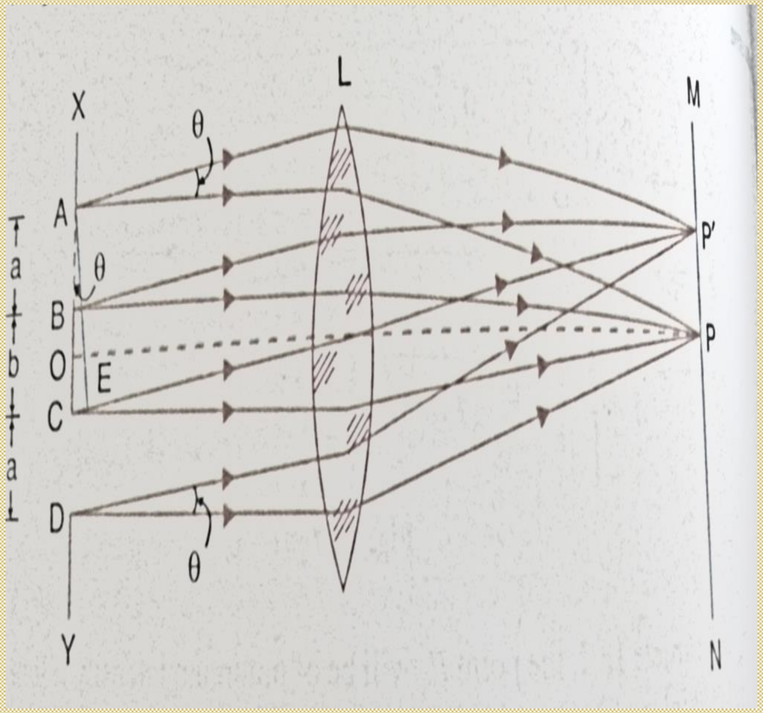
  With a narrow slit, width of central maxima is more

  Diffraction pattern consist of alternate bright and dark bands with monochromatic light.

  Central maxima is white and rest are colored with white light.

FRAUNHOFFER DIFFRACTION AT A DOUBLE SLIT:



- Diagram:
- AB &CD are two rectangular
- Slits parallel to each
  a & b are width of
  slit & opaque portion
  L is collecting length
  MN is screen

Working:

Let plane wave front incident on surface XY.

All secondary waves travelling parallel to OP come to focus at P

P point is central maxima

Diffraction pattern is due to

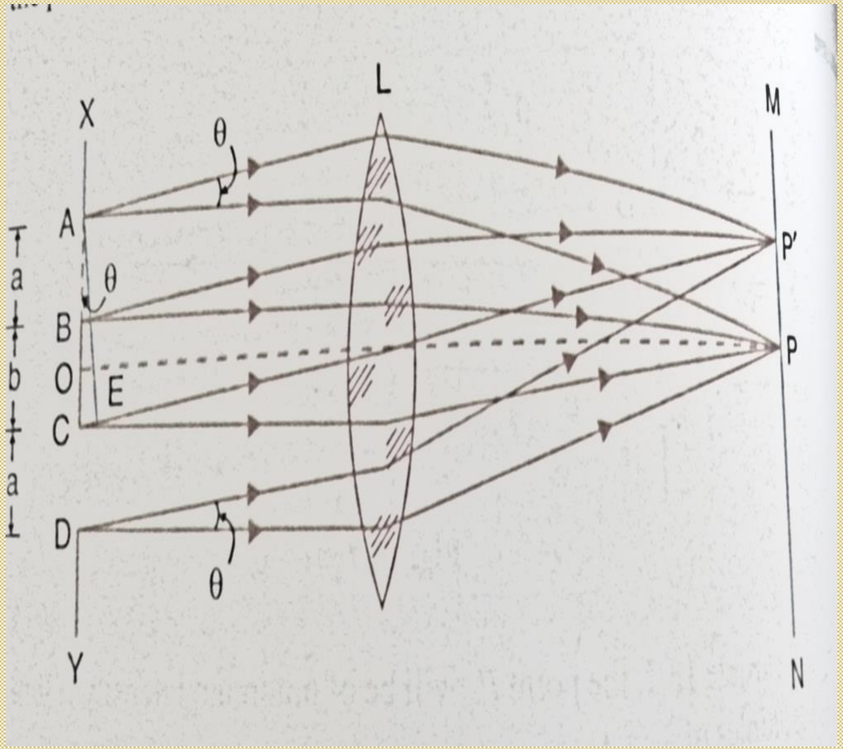
1) Interference due to waves corresponding two slit

2) Diffraction due to sec. waves from two slits indivisually

FRAUNHOFFER DIFFRACTION AT A DOUBLE SLIT:

- Secondary waves inclined θ with initial direction
  If PD BM $=\lambda$ diffraction min.
  ie. PD betn extremities of Slit $=\lambda$.
  PD betn corresponding points of upper & lower halves $=\lambda/2$

$$a \sin\theta_n = n\lambda$$



Putting n=1,2,3 etc corresponding directions of diffraction minima can be obtained

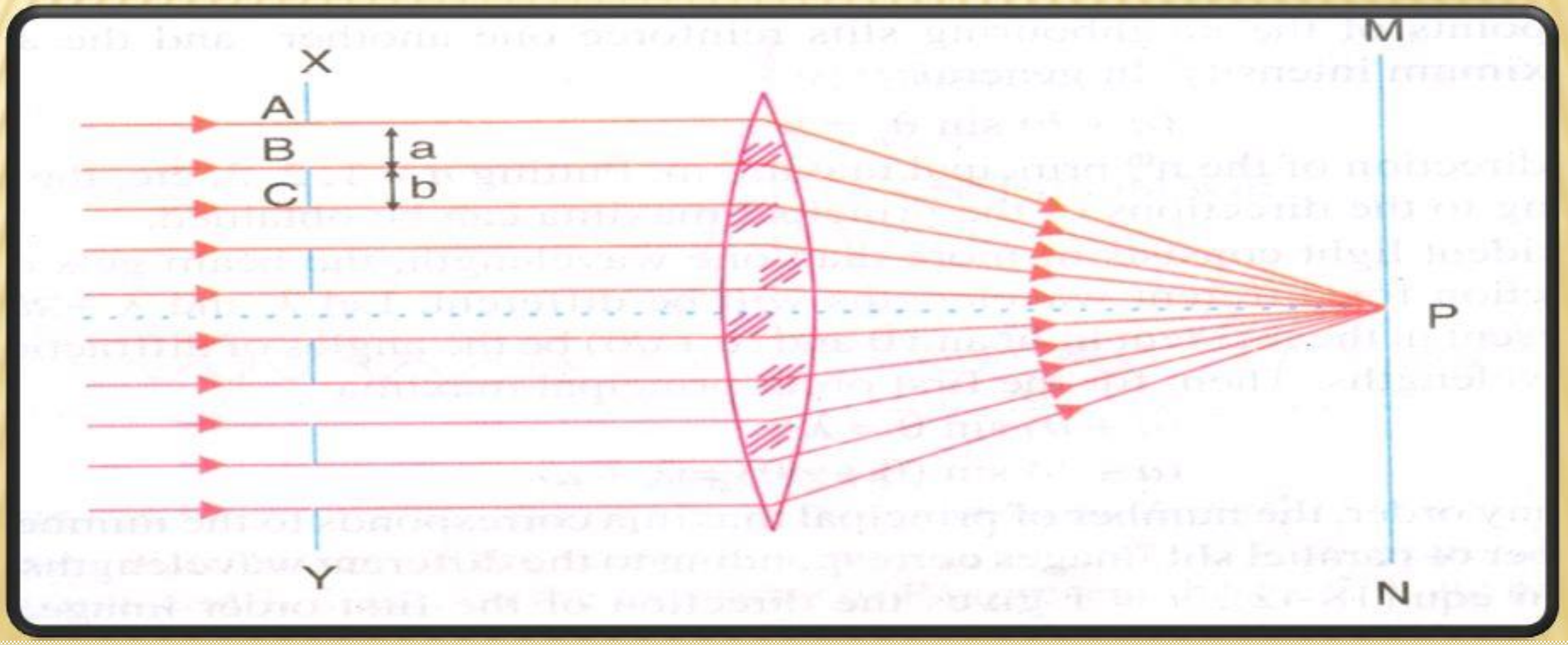
DIFFRACTION GRATING

- Diffraction Grating: An arrangement consisting of a large number of parallel slits of the same width and separated by equal opaque spaces usually made by ruling equidistant, extremely close tine grooves with a diamond point on an optically plane glass plate known as diffraction grating

PLANE DIFFRACTION GRATING

THEORY OF PLANE TRANSMISSION GRATING:

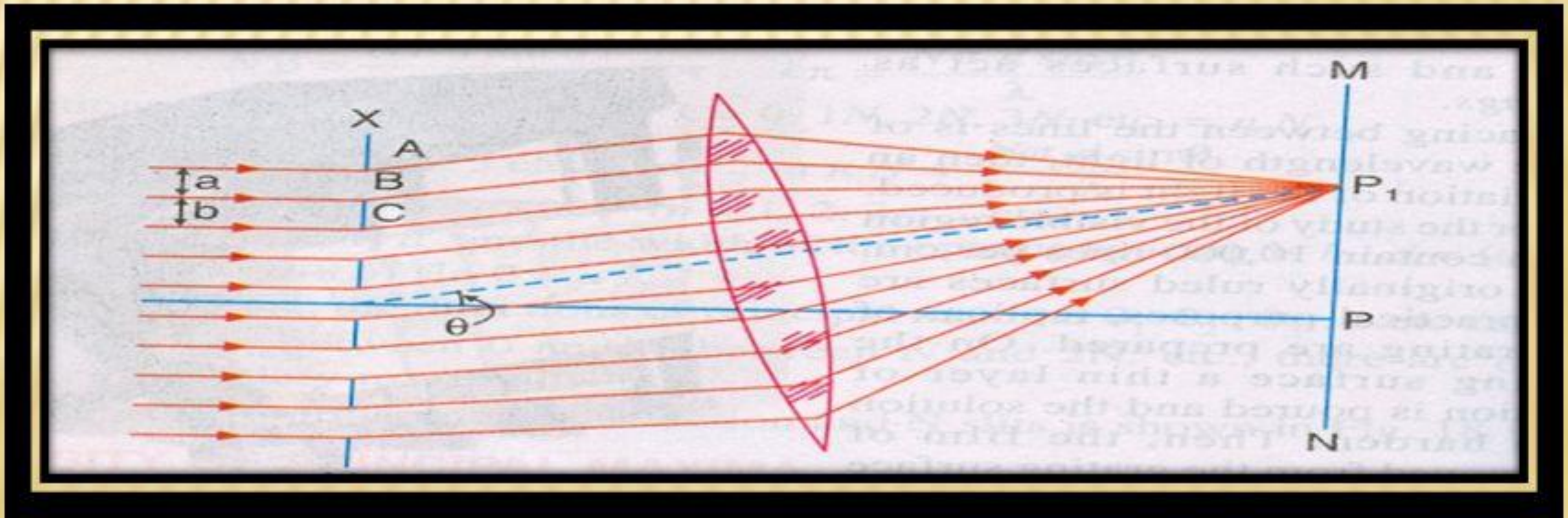
- Let XY be the grating surface and MN be the screen, both are perpendicular to the plane of paper. Here AB is the slit and BC is an opaque portion. The width of slit is *'a'* and opaque spacing between two slits is *'b'*.



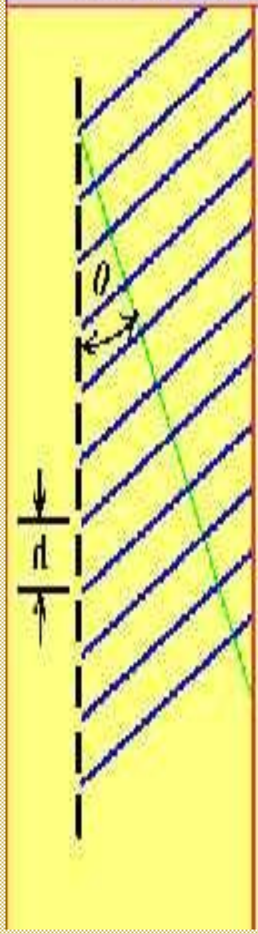
THEORY OF PLANE TRANSMISSION GRATING(contd.):

- Consider the secondary waves traveling in a direction inclined at an *angle θ* with the direction of incident light. The collecting lens also suitably rotated such that the axis of the lens is parallel to the direction of secondary waves.

- The secondary waves come to *focus at P_1*. The *intensity at P_1* will depend on path difference between the secondary waves originating from the *corresponding points A and C of two neighboring slits*.



The Diffraction Grating Explained



A **diffraction grating** is a very large number of narrow, closely-spaced slits.

When the path-length difference between paths of light from neighboring slits is an integer number of wavelengths, the light from ALL rays intefere constructively.

$$\Delta = n\,\lambda = d \sin\theta$$

For any other angle, the light from every slit will interfere destructively with the light from some other slit.

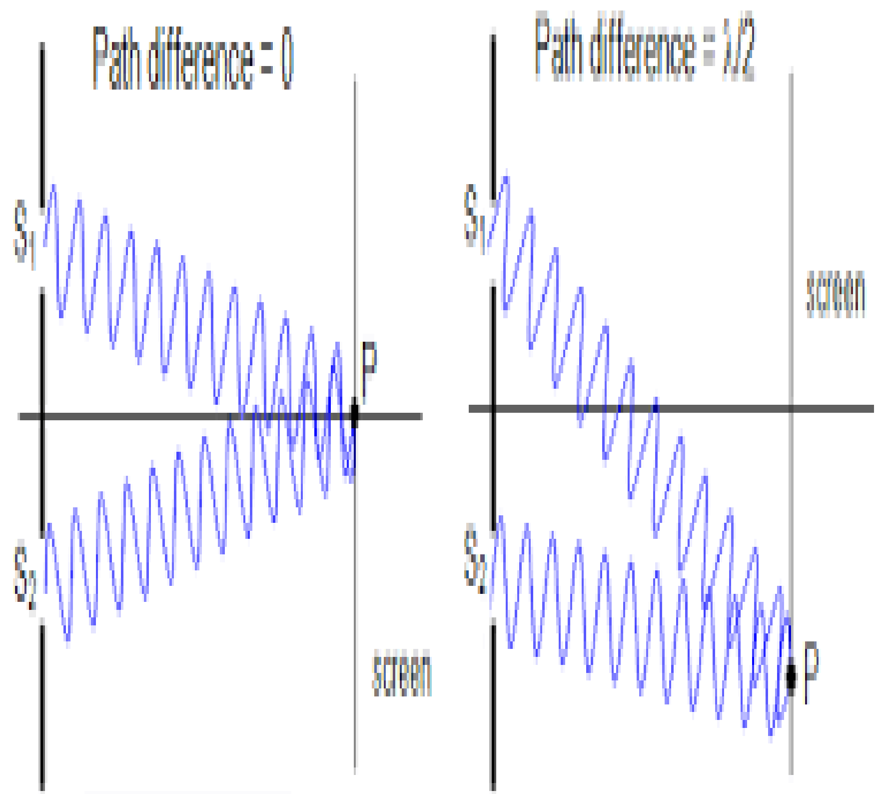


Figure 7(a)

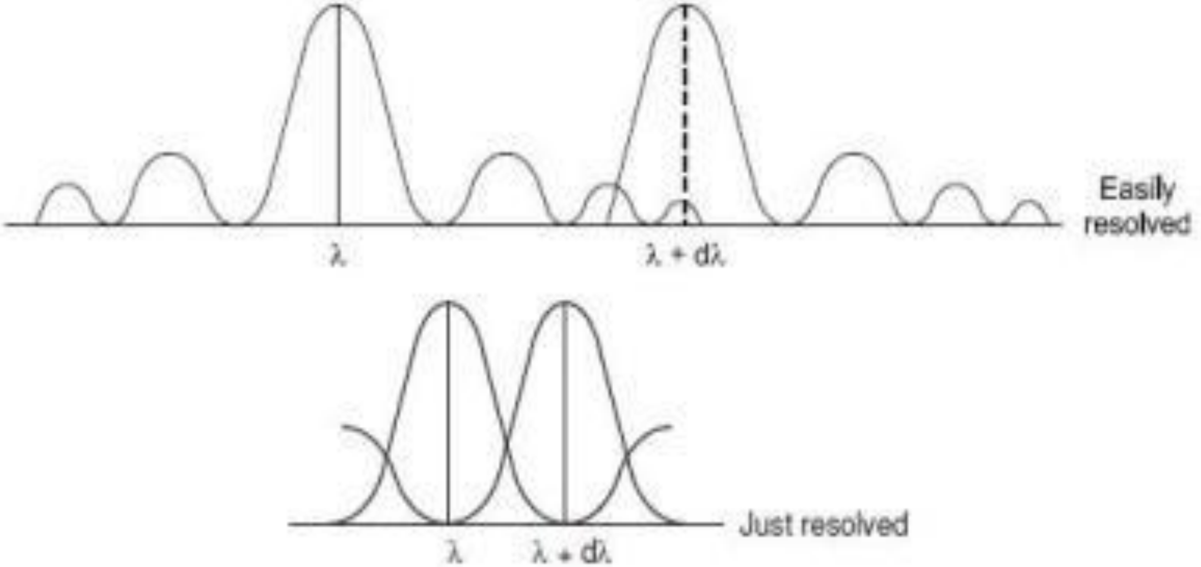
Figure 7(b)

RAYLEIGH CRITERION:

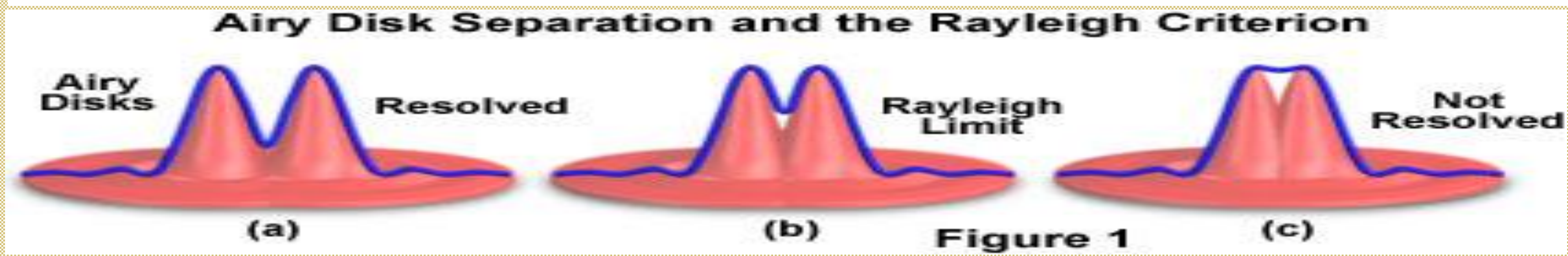
Rayleigh Criterion

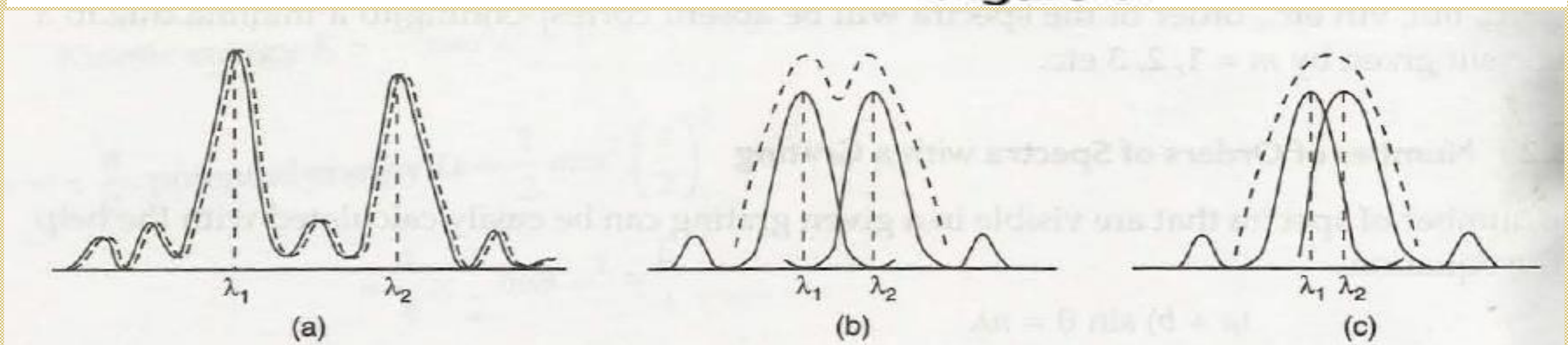
According to Rayleigh's Criterion "Two points or two spectral line of equal intensity are just resolved by an optical instrument when central maximum of the diffraction pattern due to one falls on the first minimum of diffraction pattern of the other.

Let us consider the intensity distribution curve of two wavelength λ and $\lambda + d\lambda$. If $d\lambda$ is larger then the two spectral lines are easily resolvable



RAYLEIGH CRITERION:





RESOLVING POWER:

- The capacity of an optical instrument to show separate images of very closely placed two objects is called **resolving power**.
- Resolving power is defined as the reciprocal of smallest angle subtended at the objective of an optical instrument by two point object, which can just be distinguished as separate

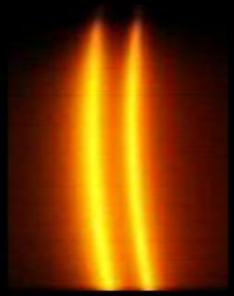
Resolving power of a diffraction grating

- The **resolving power** of a diffraction **grating** is defined as its ability to form separate diffraction maxima of two closely separated wave lengths.

Resolving power of diffraction grating

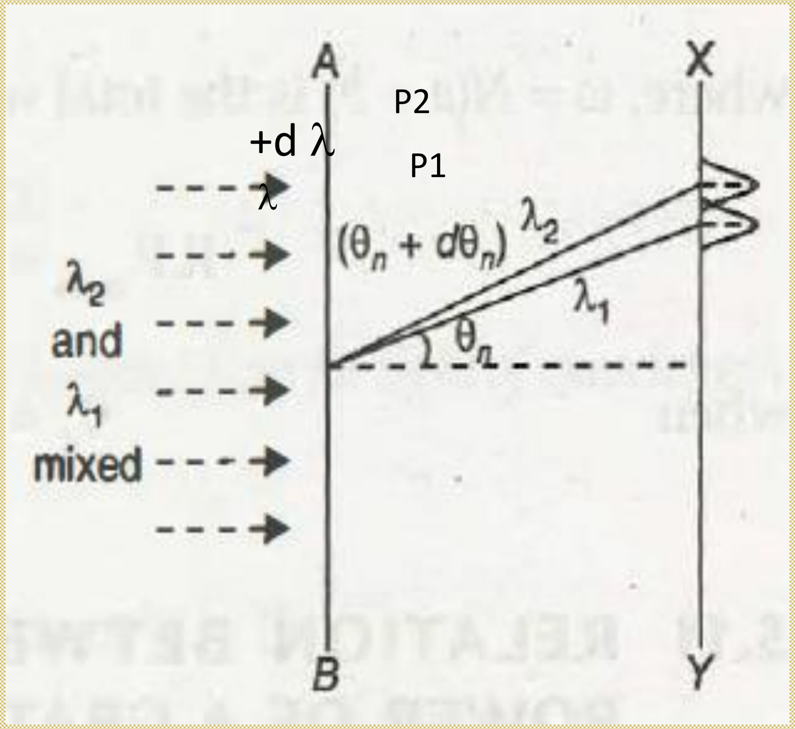
The R.P. of grating is defined as the, ratio of wavelength λ of any spectral line to the smallest difference in wavelength dλ, between this line and a neighboring line such that the two lines appear just resolved, according to Rayleigh's criterion.

$$\text{R.P. of grating} = \frac{\lambda}{d\lambda}$$



Resolving power of a diffraction grating

- AB-- Grating surface
- XY– Field of view of telescope
- P1 –nth primary maxima of spectral line of wavelength λ at θ
- P2 –nth primary maxima
- of spectral line of
- wavelength $\lambda+d\lambda$ at
- $(\theta+d\theta)$
- According to Rayleigh
- Two spectral lines just
- resolved if position of P2
- corresponds to first minimum of P1



Resolving power of a diffraction grating

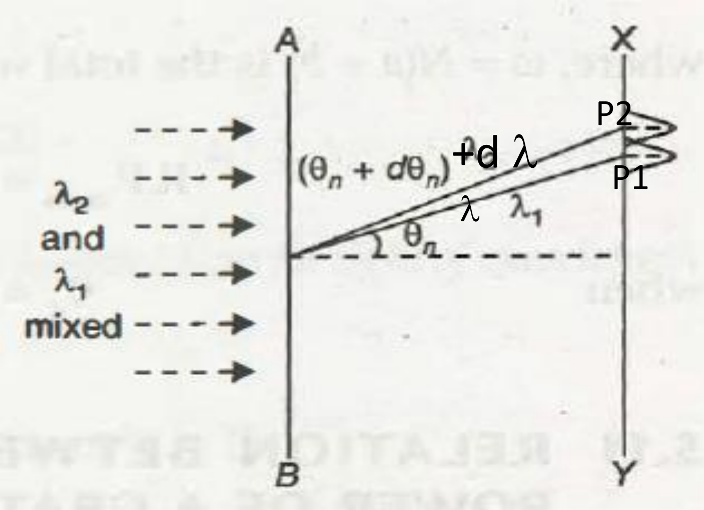
- Direction of n th primary maximum for λ is

$$(a+b)\sin\theta_n = n\lambda \qquad (1)$$

- Direction of n th primary maximum for λ+dλ is

$$(a+b)\sin(\theta_n + d\theta) = n(\lambda + d\lambda) \qquad (2)$$

- The two lines will just
- Resolved if extra path
- Difference of λ/N is
- Introduced



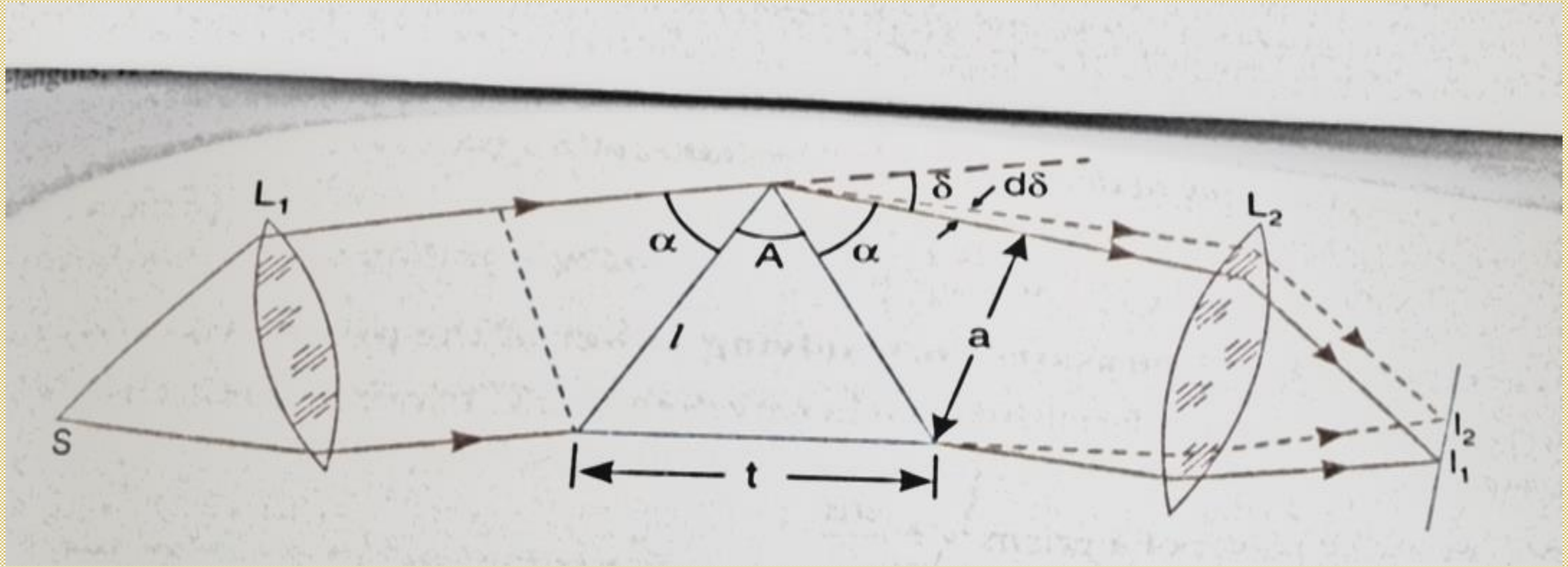
$$(a+b)\sin(\theta_n + d\theta) = n(\lambda + d\lambda) = n\lambda + \lambda/N$$

$$n(\lambda + d\lambda) = n\lambda + \frac{\lambda}{N}$$

$$nd\lambda = \frac{\lambda}{N}$$

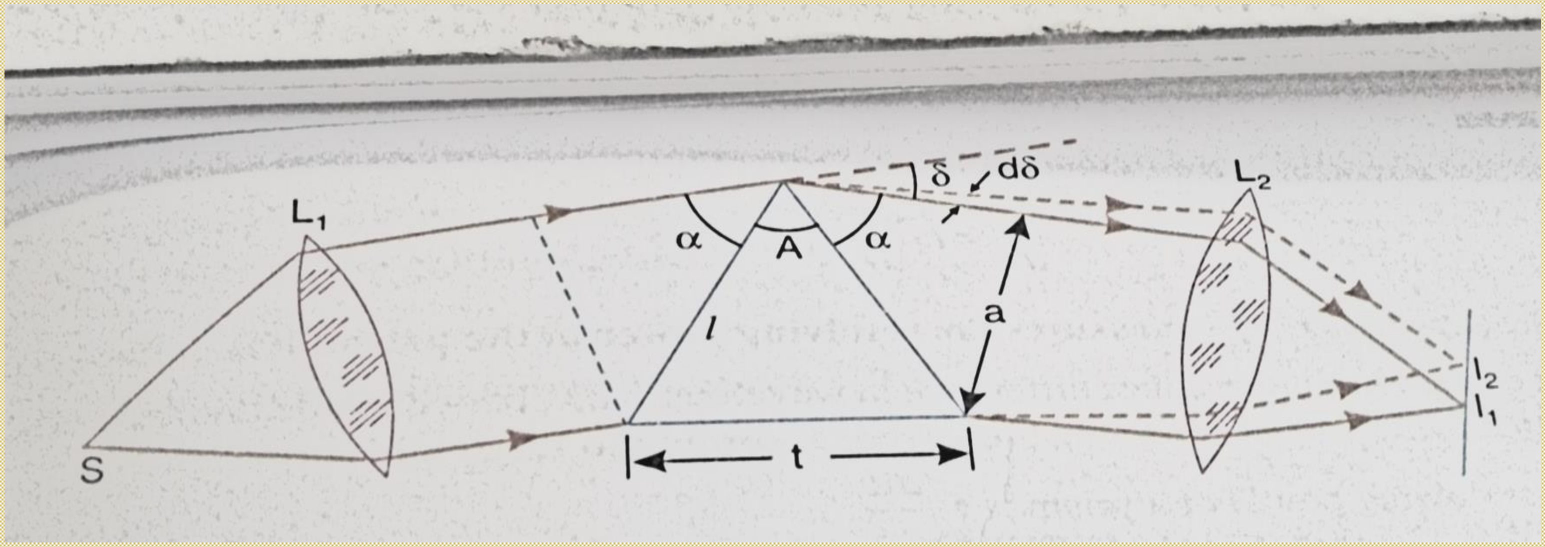
$$\frac{\lambda}{d\lambda} = nN$$

R.P. OF PRISM



- S –the source of
- L1- Collimating lens
- L2- Telescope objective
- Two wavelength λ and $d\lambda$ are very close
- Prism is set into minimum deviation position for both
- L1 –corresponds to principal maxima for λ
- L2–corresponds to principal maxima for $\lambda + d\lambda$

R.P. OF PRISM



- Face of prism limits incident beam to a rectangular section of width “a”.
- Applying Rayleigh’s criteria for rectangular aperture
- The position of I2 corresponds to first minimum of I1 for λ provided $a\,d\delta = \lambda \;\; or \; d\delta = \dfrac{\lambda}{a} - - - - - - (i)$

$$From\ fig \quad \alpha + A + \alpha + \delta = \pi$$

$$\therefore \alpha = \left[\left(\frac{\pi}{2}\right) - \left(\frac{A+\delta}{2}\right)\right] \qquad \therefore \sin\alpha = \left[\left(\frac{\pi}{2}\right) - \left(\frac{A+\delta}{2}\right)\right] = \cos\left(\frac{A+\delta}{2}\right)$$

$$\sin\alpha = \frac{a}{l}$$

$$\therefore \cos\left(\frac{A+\delta}{2}\right) = \frac{a}{l}$$

$$\therefore \sin\alpha = \left[\left(\frac{\pi}{2}\right) - \left(\frac{A+\delta}{2}\right)\right]$$

$$also \sin\frac{A}{2} = \frac{t}{2l}$$

- In the case of prism

$$\mu = \frac{sin\frac{A+\delta}{2}}{sin\frac{A}{2}}$$

$$\therefore \; sin\frac{A+\delta}{2} = \mu\, sin\frac{A}{2}$$

- μ and δ are dependent on wavelength λ
- Differentiating above equation

$$\frac{1}{2}cos\left(\frac{A+\delta}{2}\right)\frac{d\delta}{d\lambda} = \frac{d\mu}{d\lambda}\left(sin\frac{A}{2}\right)$$

- Substituting values of dδ

$$\therefore \frac{1}{2}\left(\frac{a}{l}\right)\frac{d\delta}{d\lambda} = \frac{d\mu}{d\lambda}\left(\frac{t}{2l}\right)$$

$$a\frac{d\delta}{d\lambda} = t\ \frac{d\mu}{d\lambda}$$

$$\frac{\lambda}{d\lambda} = t\frac{d\mu}{d\lambda}$$

R.P. of Prism= $t\frac{d\mu}{d\lambda}$