UNIT: I
CARDINAL POINTS

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CARDINAL POINTS OF AN OPTICAL SYSTEM:

Definition of a co-axial optical system:

A combination two or more lenses having a common principal axis on which the centre of curvature of all the spherical surfaces lie is called coaxial system of lenses.

Cardinal points:

since a thin lens consists of two refracting surfaces a coaxial system will have a large number of refracting surfaces. In order to determine the size and position of the image, one has to consider the refraction at each surface separately. It is very tedious to the determine the size and position of the image in a coaxial system as well as in the case of a thick lens. In order to overcome this difficulty six cardinal points of an optical system were suggested.

CARDINAL POINTS OF AN OPTICAL SYSTEM:

Equivalent Lens:

A single lens is said to be equivalent to a number of lenses when this single object able to produce an image of a given object at the same place and of the same size as that formed by a number of lenses together. The foal length of such a single lens is termed as the equivalent focal length which is equal to the focal length of combination of lenses.

SIX CARDINAL POINTS:

- **CONJUGATE POINTS:**

It is known that as a result of refractions (and reflections) of the rays from a point object A in a given space there corresponds a point image B. Points A and B are called conjugate points.

CONJUGATE PLANES:

If two rays from the object interesected at the point object A, the two conjugate refracted rays intersect at the point image B. Since a plane is defined by two rays, therefore to every plane in the object space there corresponds a conjugate plane in the image space. To sum up, to every point, ray or plane in the object spaces, there corresponds one, and only one, point, ray or plane in the image space

SIX CARDINAL POINTS:

Cardinal points (or Gauss points):

In a coaxial system, there are six cardinal points which greatly simply the study of the formation of images and the tracing of conjugate rays.

These points are purely theoretical earlier deduced by Gauss With the help of these points

The position and size of image of an object may then directly be determined by the simple formulae for thin lens

SIX CARDINAL POINTS:

(1) Two focal points
(2) Two principal points
(3) Two nodal point.

Cardinal planes:
The planes passing through cardinal points and perpendicular to principal axis

The cardinal points and planes are the intrinsic properties of an optical system and determine image forming properties

PRINCIPAL POINTS AND PRINCIPAL PLANES:

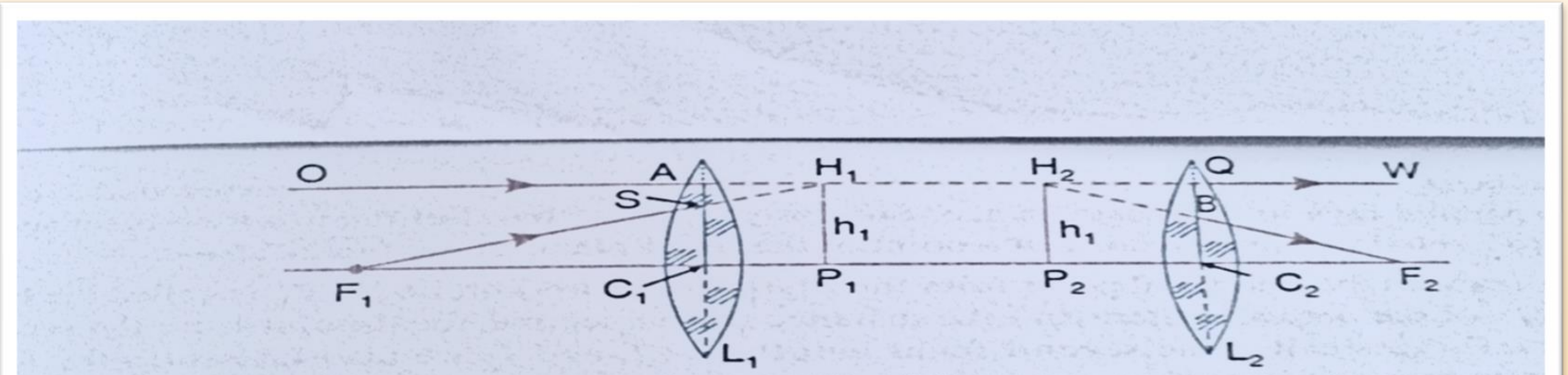


Fig. 5.2

- F1 & F2 are Focal points
- A ray OA travelling parallel to principal axis incident at A is brought to focus at F2 in the image space
- At each surface the ray is refracted and follow the path OABF2
- If we extend the incident ray OA forward and emergent ray BF2 backward ,they meet each other within the optical system at H2

PRINCIPAL POINTS AND PRINCIPAL PLANES:

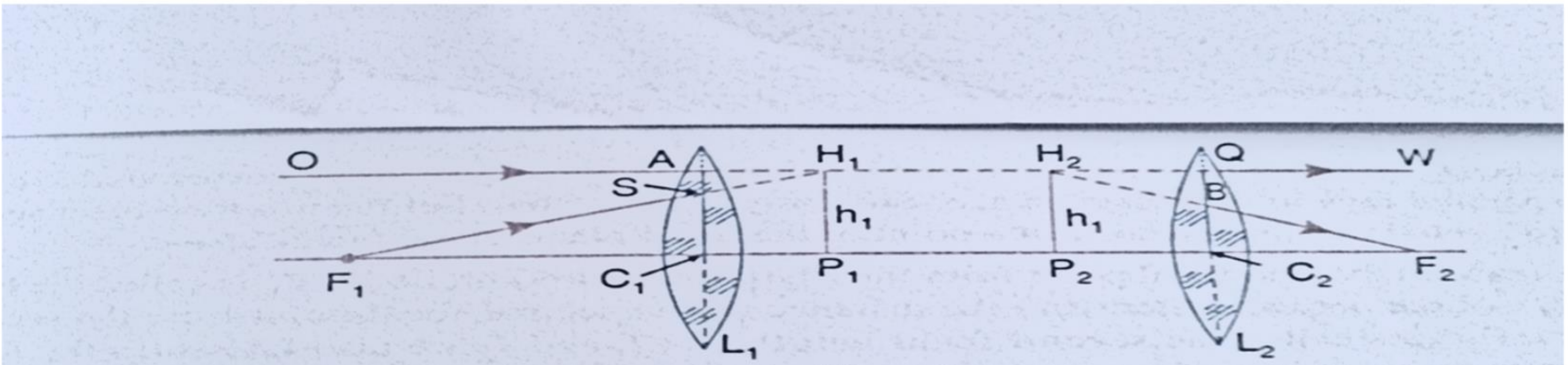


Fig. 5.2

- Plane passing through H2 and perpendicular to axis is the surface at which refraction takes place. This plane is called **PRINCIPAL PLANE**
- Principal plane is defined as the loci where refraction appears to be occurs without any reference to actual refraction.

PRINCIPAL POINTS AND PRINCIPAL PLANES:

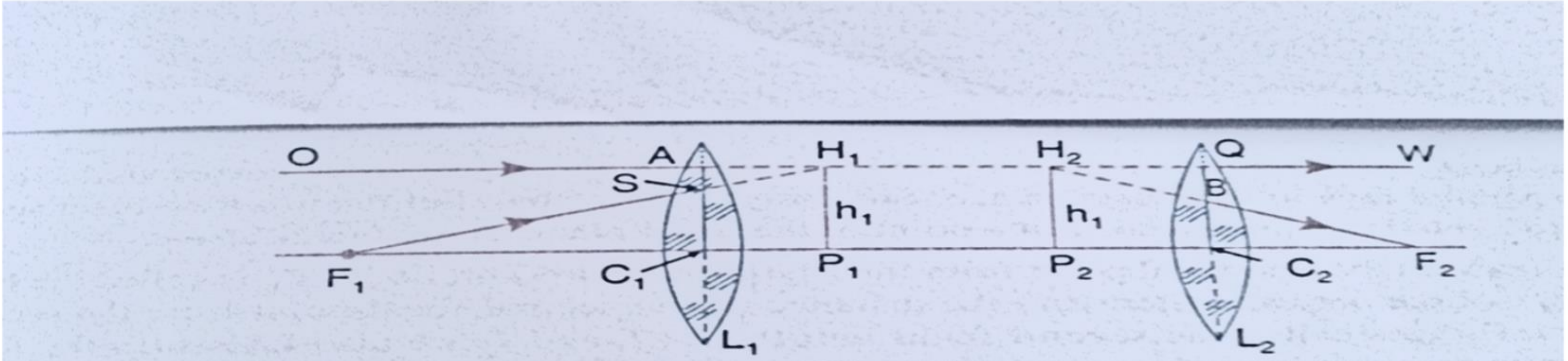


Fig. 5.2

The second principal plane in the image space is the locus of the points of intersection of the incident rays (in the object space) parallel to the axis and their conjugate emergent rays (in the image space).

The intersection of the second principal plane with the axis is called the second principal point (P2). The distance P2F2 is called the second focal length.

PRINCIPAL POINTS AND PRINCIPAL PLANES:

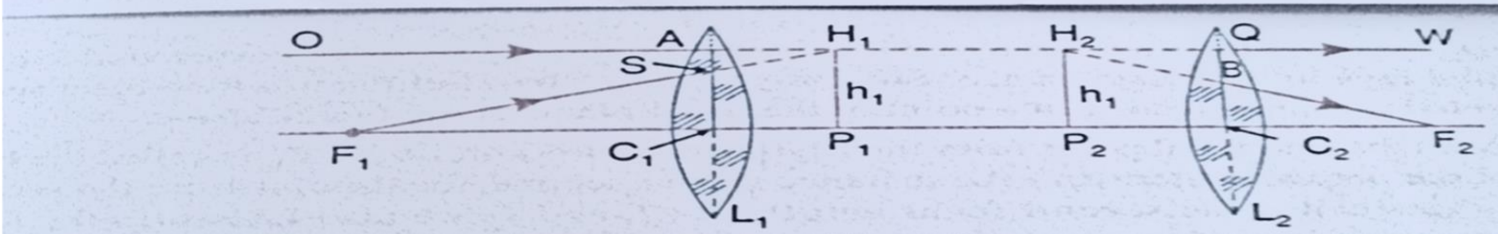
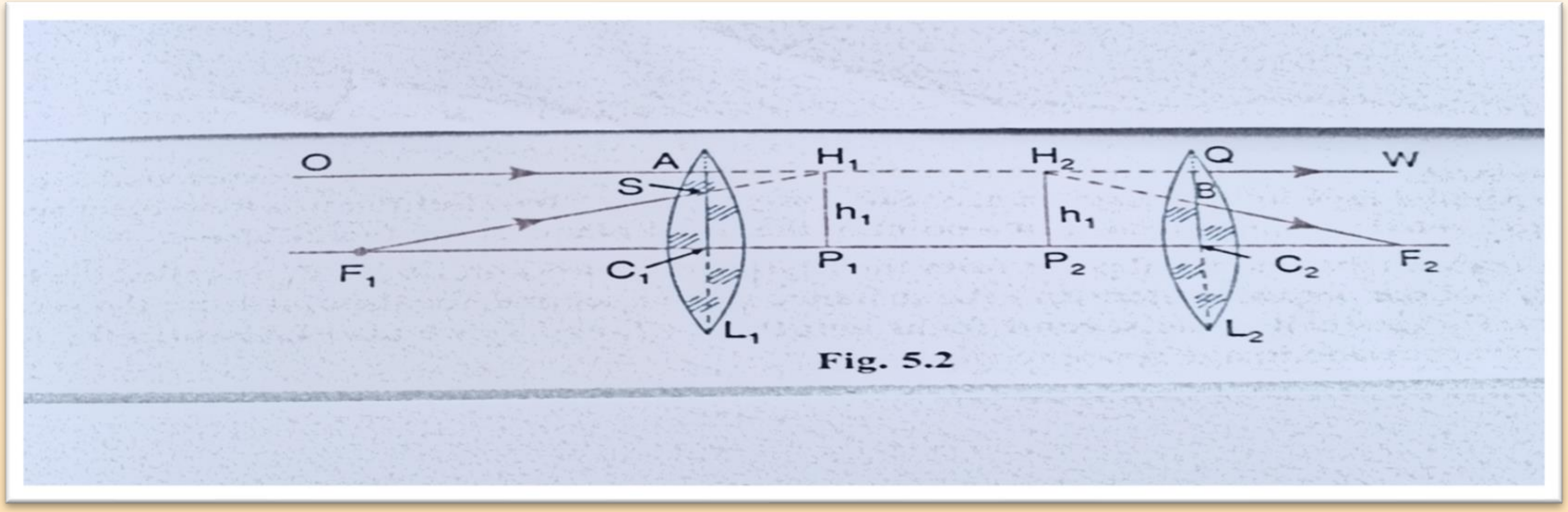


Fig. 5.2

- Similarly ray F_1S passing through first principal focus F_1 such that after refraction it emerges along QW parallel to axis at same height as that of OA
- The rays F_1S and QW intersect at H_1.
- The plane perpendicular to axis and passing through H1 is called First principal plane and point of intersection is called First principal point

PRINCIPAL POINTS AND PRINCIPAL PLANES:

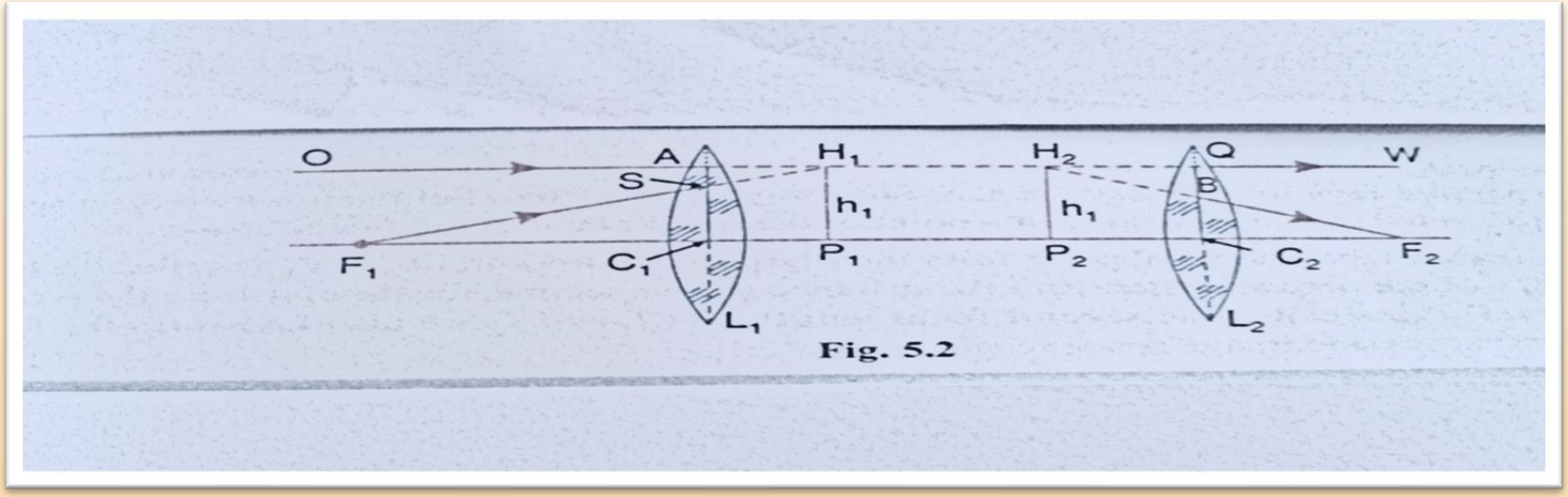


The first principal plane in the object space is the locus of the points of intersection of the emergent rays in the images space parallel to the axis and their conjugate incident rays in the object space.

The intersection of the first principal plane with the axis is called the first principal point (P1).

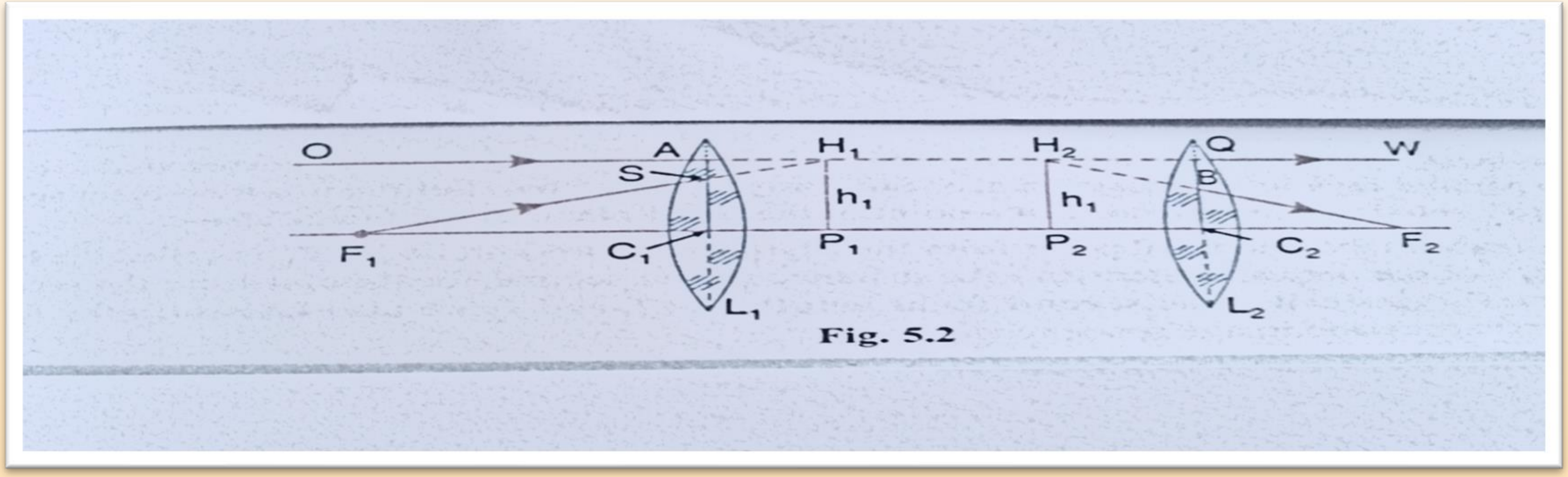
The distance F1P1=f1, where f1 is the first focal length of the coaxial system.

PRINCIPAL POINTS AND PRINCIPAL PLANES:



- From fig the two incident rays directed towards H_1 and after refraction seems to come from H_2.Therefore H_2 is image of H_1
- H_1 and H_2 are conjugates points H_1P_1 and H_2P_2 are pair of conjugate pairs
- $H_2P_2=H_1P_1$

FOCAL POINTS AND FOCAL PLANES:



- The first focal point: is the point on the principal axis of the optical system such that a beam of light passing through it is rendered parallel to principal axis after refraction through the optical system.
- The second focal point: is the point on the principal axis of the optical system such that a beam of light travelling parallel to principal axis of the optical system, after refraction through the system, passing through it

FOCAL POINTS AND FOCAL PLANES:

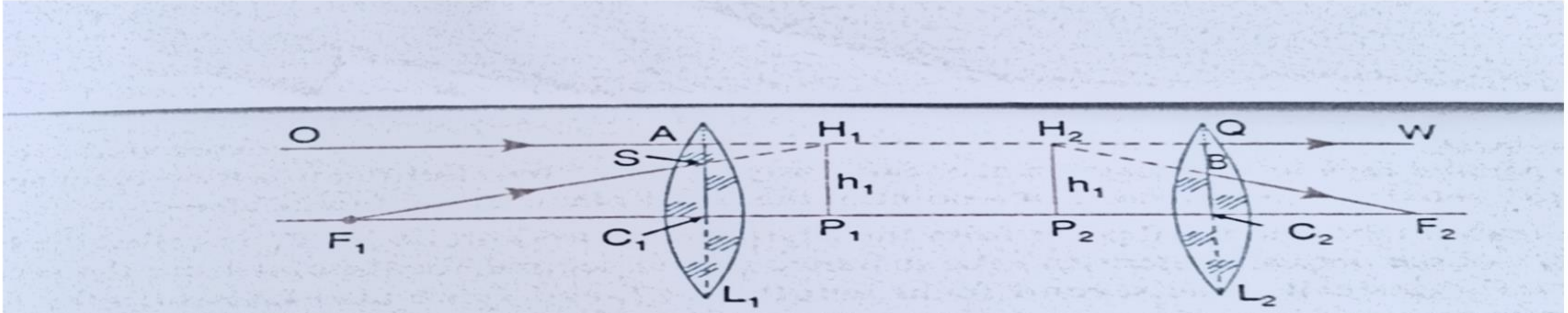
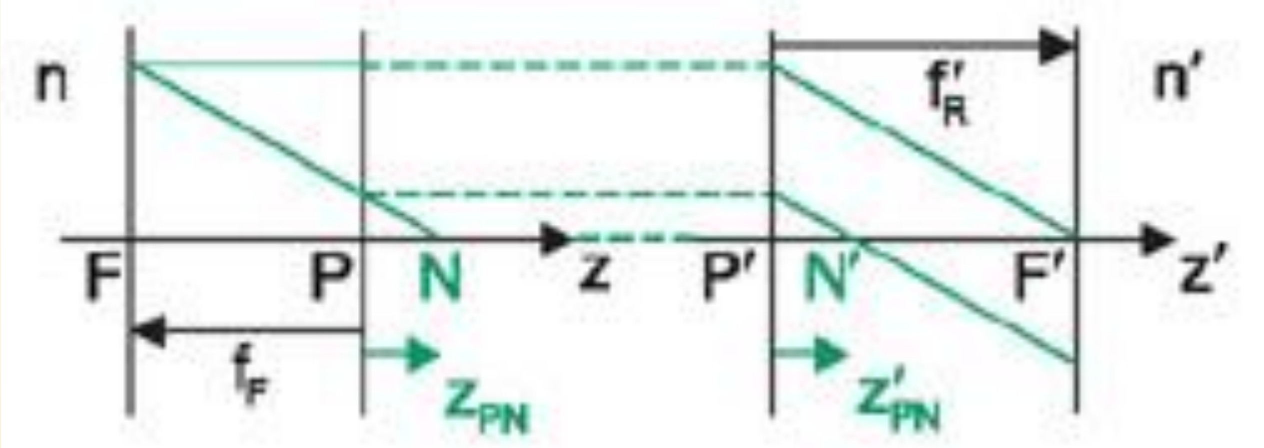


Fig. 5.2

- The planes passing through the focal points and perpendicular to the axis called focal planes.
- First focal length: The distance of the first focal point from the first principal plane F_1P_1 is called First focal length
- Second focal length: The distance of the second focal point from the second principal plane F_2P_2 is called First focal length
- When medium is same on two sides of optical system then $f_1=f_2$

NODAL POINTS AND NODAL PLANES:



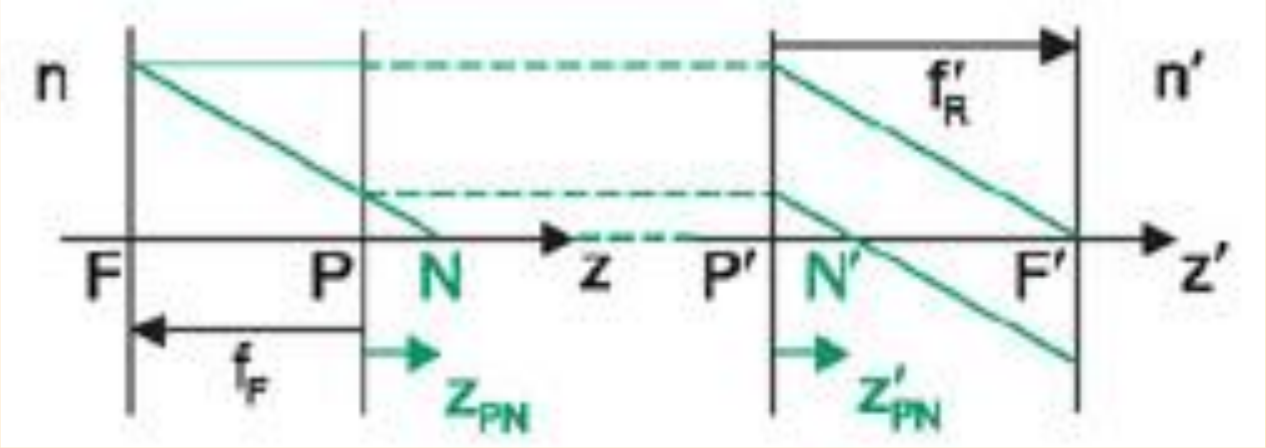
Nodal points are the points on principal axis of the optical system where light rays,without refraction, intersect the optical axis

In a thin lens nodal point is the centre of the lens. Light passing through the centre of thin lens does not deviates

Nodal Planes: The planes passing through the nodal points and perpendicular to the principal axis

Nodal planes are planes where refraction does not take place

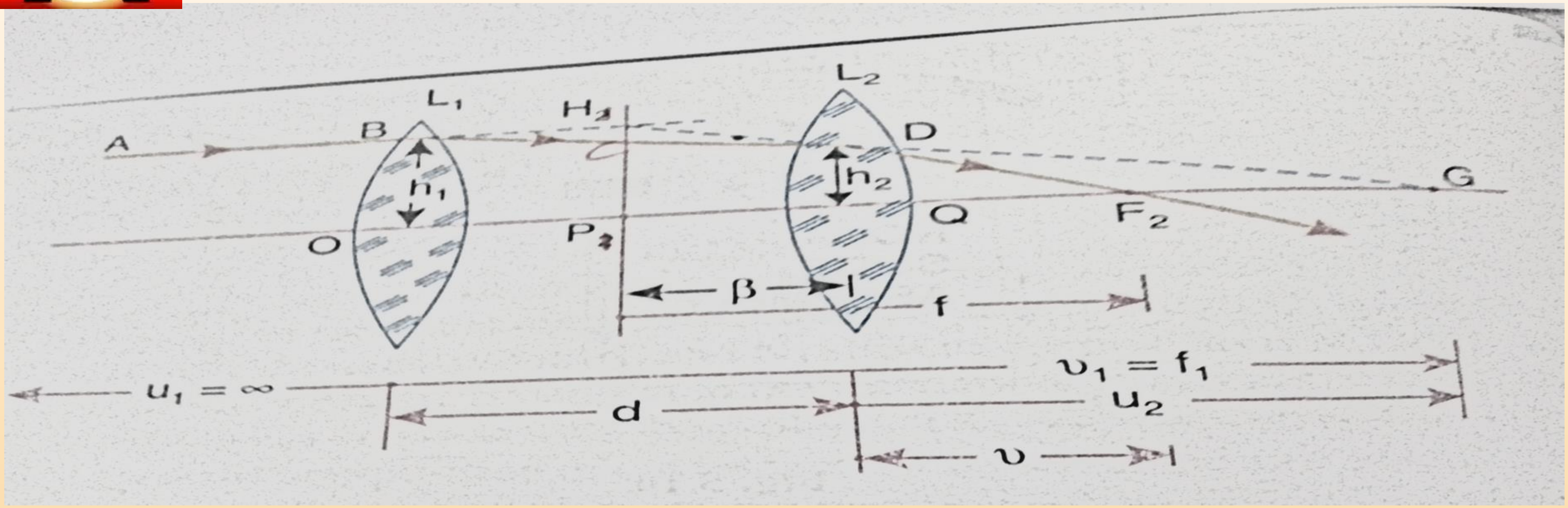
NODAL POINTS AND NODAL PLANES:



From fig. A ray of light Nn,directed towards N nodal point ,after refraction through the optical system, along NN’, emerges out from the second nodal point N’ in a direction parallel to incident ray

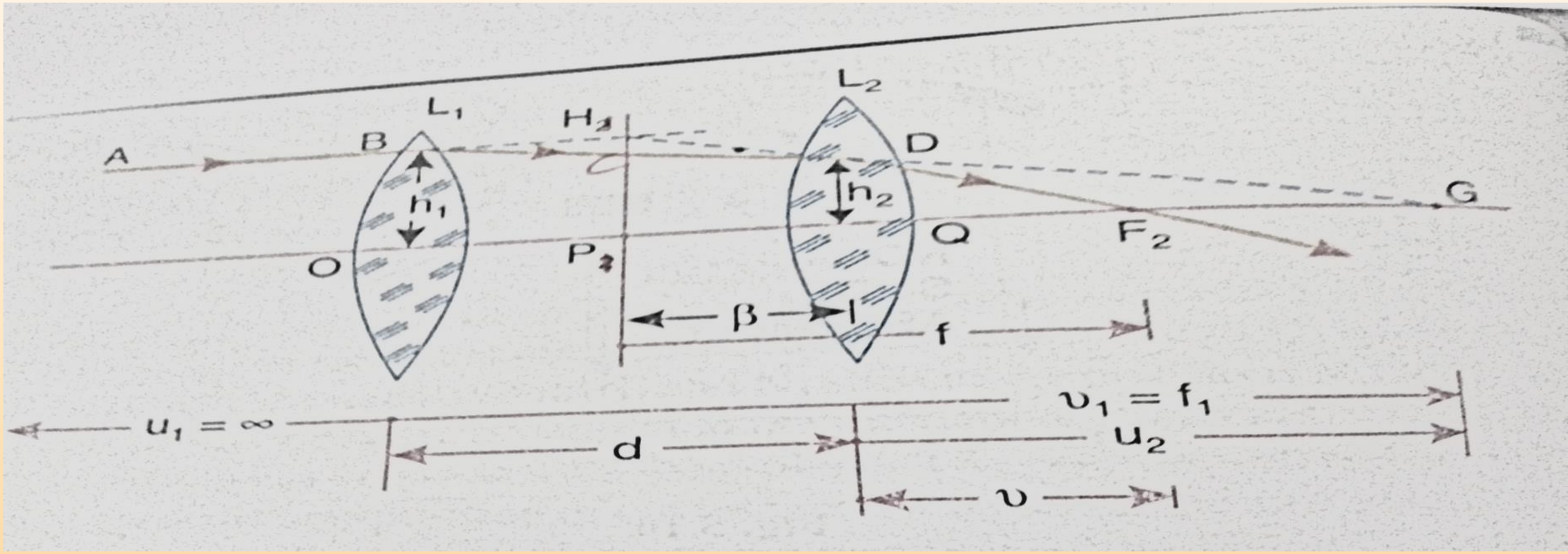
Distance of the nodal points were measured from focal points

CARDINAL POINTS OF A COAXIALSYSTEM OF TWO THIN LENSES:



Ray AB coming from an object at infinity(ie $\mu_1=\infty$)
If L_1 is alone, image would be at G. Because of L_2, G becomes the virtual object for L_2
Ray BD,instead of going along BDG, refract along the path DF_2
When AB is produced forward and DF_2 backward intersect at H_2.
Plane H_2P_2 normal to axis and considered refraction to occurs known as PRINCIPAL PLANE

CARDINAL POINTS OF A COAXIALSYSTEM OF TWO THIN LENSES:

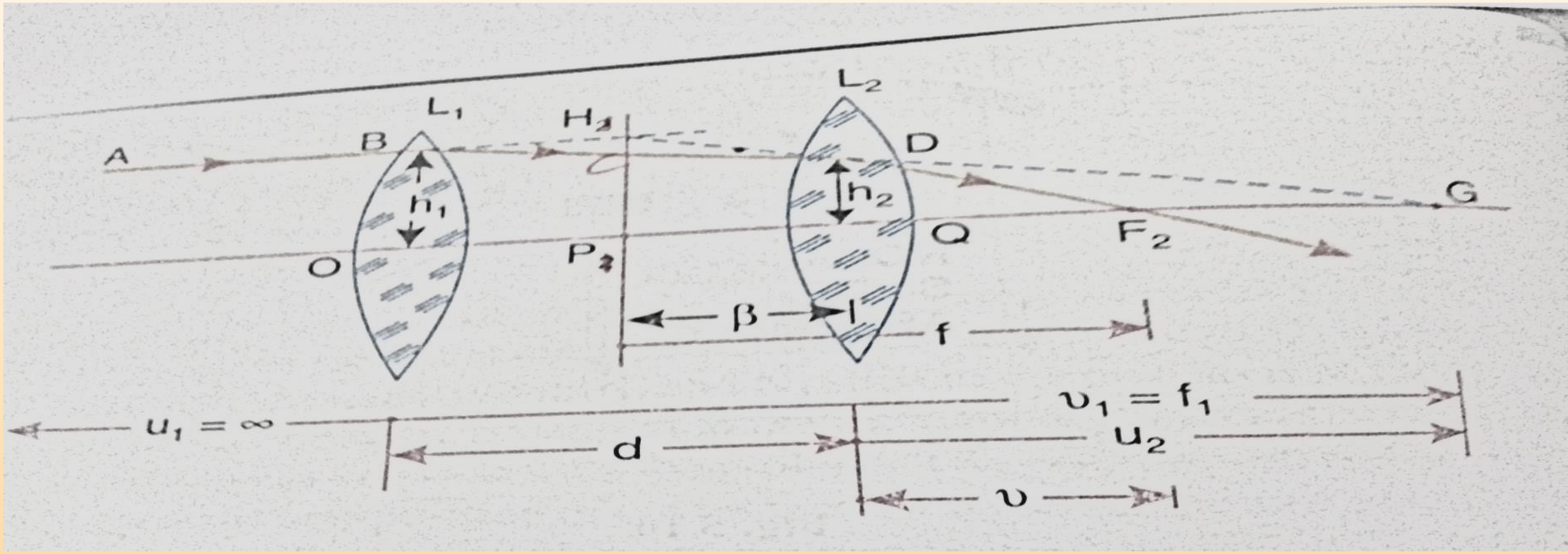


Expression for refraction at first lens

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1} \qquad \therefore \frac{1}{OG} - \frac{1}{u_1} = \frac{1}{f_1}$$

As $u_1 = \infty$ we obtain $OG = f_1$

CARDINAL POINTS OF A COAXIALSYSTEM OF TWO THIN LENSES:



Expression for refraction at second lens

$$\therefore \frac{1}{v} - \frac{1}{u_2} = \frac{1}{f_2}$$

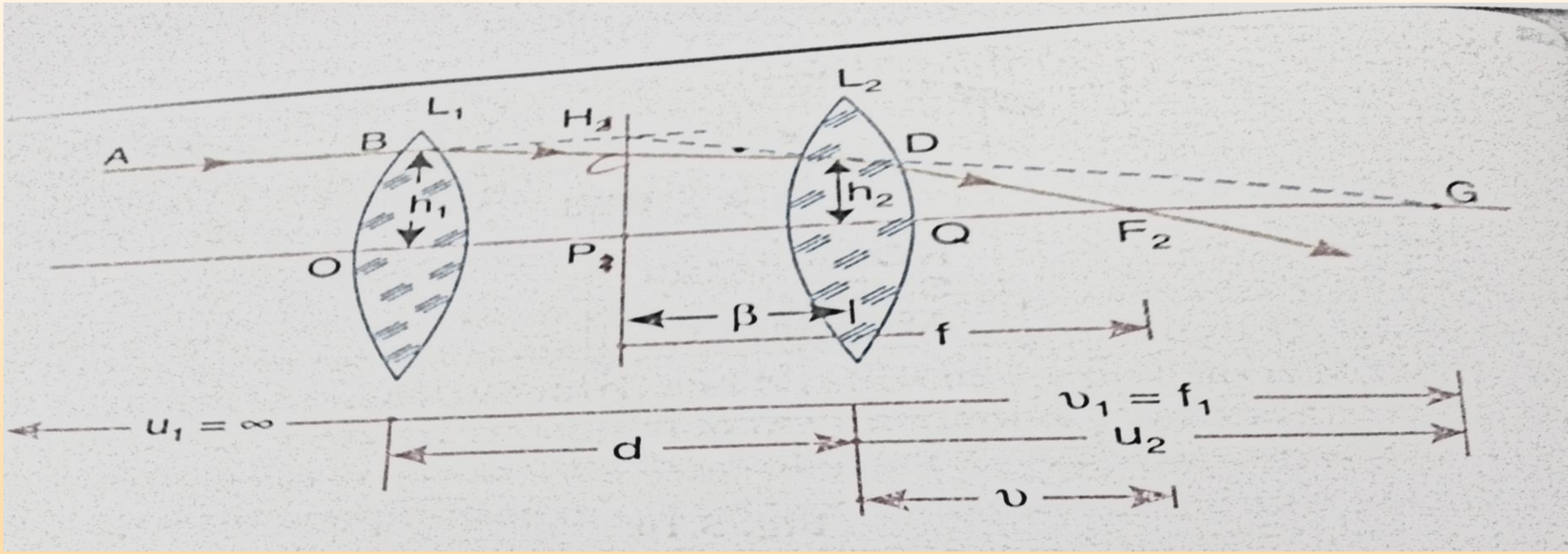
$$\therefore \frac{1}{QF2} - \frac{1}{QG} = \frac{1}{f_2}$$

$$\frac{1}{QF2} = \frac{1}{f_2} + \frac{1}{f_1 - d}$$

$$\therefore \frac{1}{QF2} = \frac{f_1 + f_2 - d}{f_2(f_1 - d)}$$

Eq.1

CARDINAL POINTS OF A COAXIALSYSTEM OF TWO THIN LENSES:

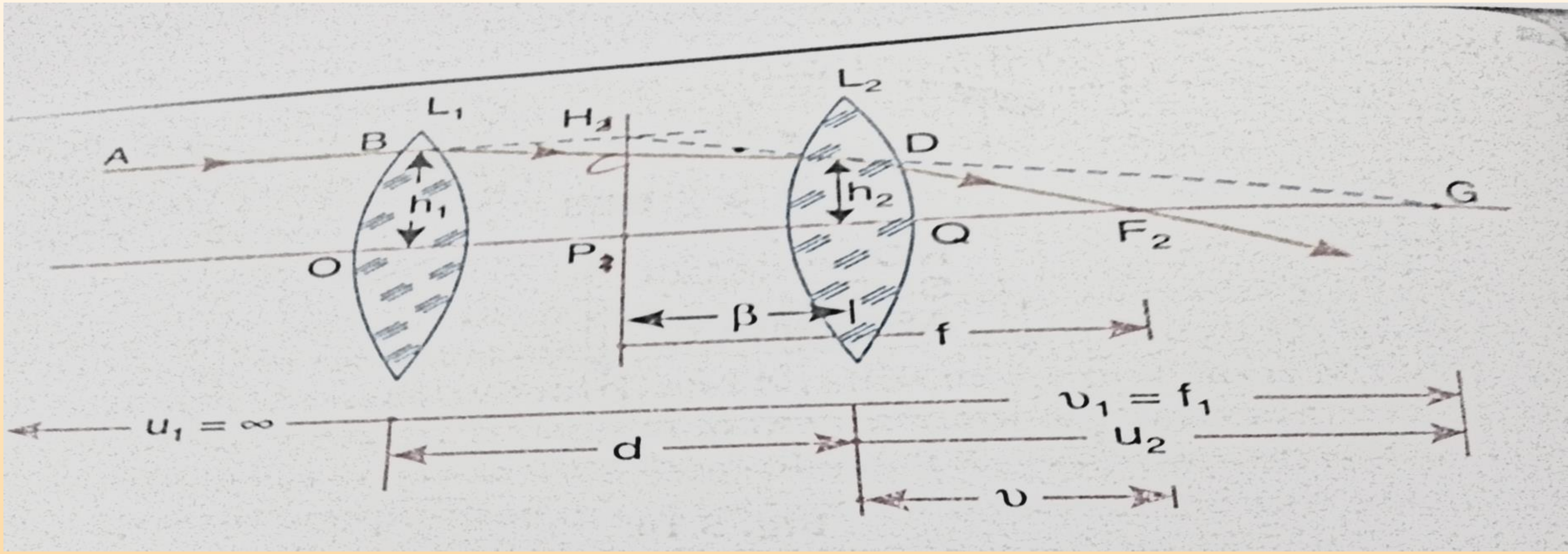


ΔBOG and DQG are similar and also $H_1P_1F_2$ and DQF_2 are similar

$$\frac{BO}{OG} = \frac{DQ}{QG} \qquad \frac{h_1}{f_1} = \frac{h_2}{(f_1 - d)} \qquad \text{Eq.2}$$

$$\frac{H_1P_1}{P_1F_2} = \frac{DQ}{QF_2} \qquad \therefore \frac{h_1}{f} = \frac{h_2}{QF_2} \qquad \text{Eq.3}$$

CARDINAL POINTS OF A COAXIALSYSTEM OF TWO THIN LENSES:

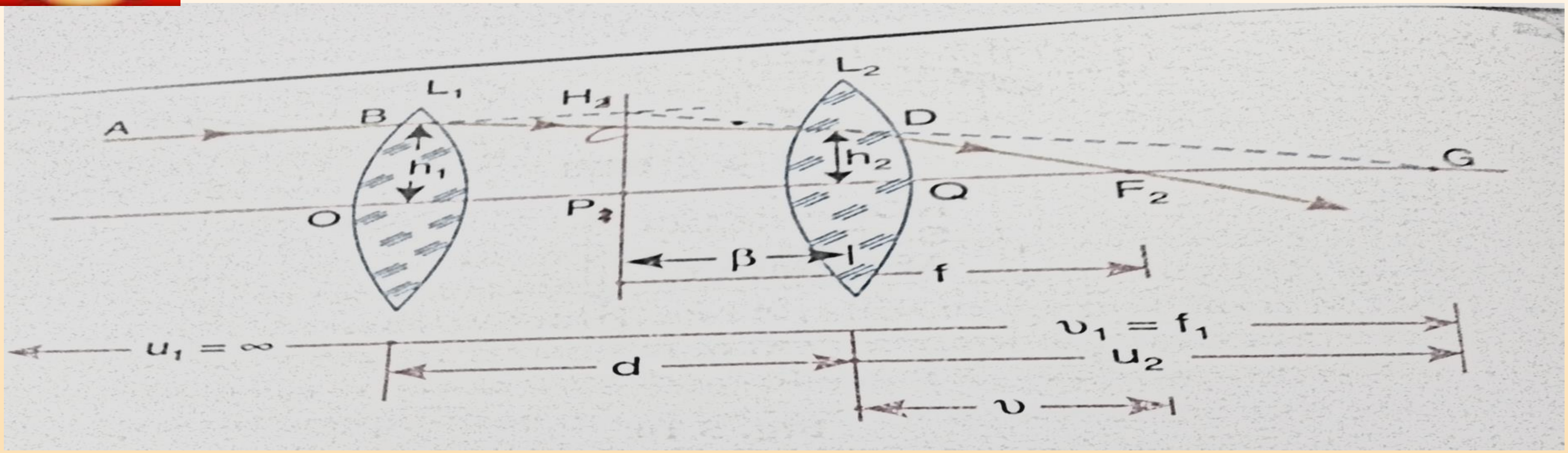


Eq.2 eq.3 and eq.1

$$\frac{h_1}{h_2} = \frac{f_1}{-(f_1 - d)} = \frac{f(f_1 + f_2 - d)}{f_2(f_1 - d)} \qquad or \quad \frac{1}{f} == \frac{f_1 + f_2 - d}{f_1 f_2}$$

$$or \quad \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

CARDINAL POINTS OF A COAXIALSYSTEM OF TWO THIN LENSES:



i) Second principal points:

Let the second principal plane is located at a distance of β from second lens L2

β would be negative

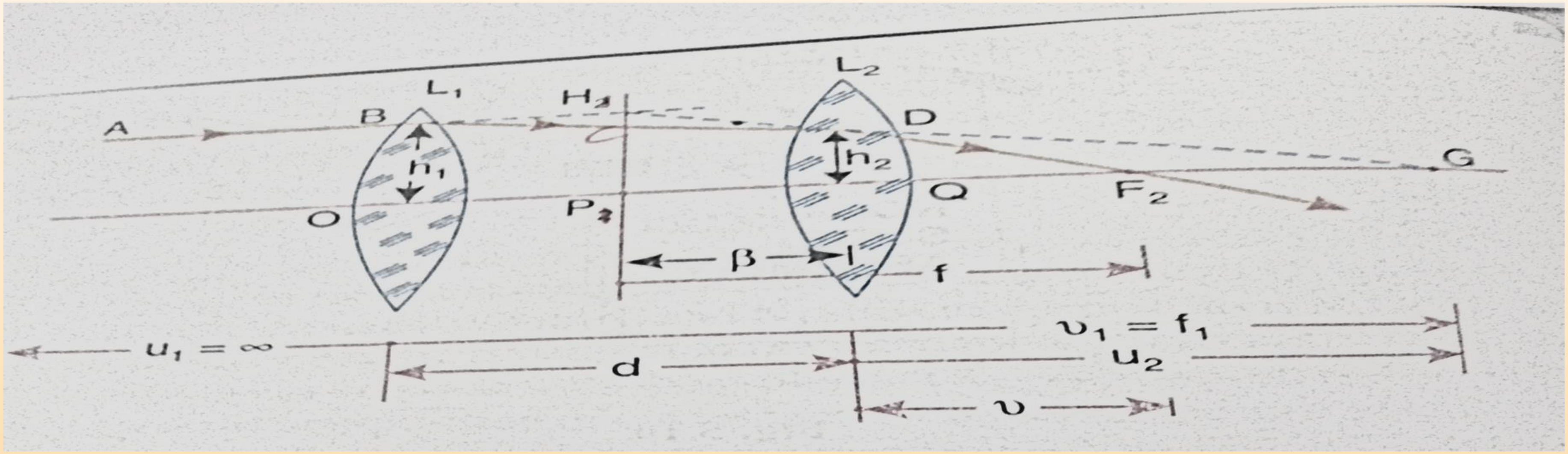
$$QF_2 = f - (-\beta) = f + \beta \qquad f + \beta = \frac{f_2(f_1 - d)}{f_1 + f_2 - d} \qquad \beta = -f + \frac{f_2(f_1 - d)}{f_1 + f_2 - d}$$

$$\beta = \frac{-f_1 f_2}{f_1 + f_2 - d} + \frac{f_2 f_1 - f_2 d}{f_1 + f_2 - d} \qquad \beta = \frac{f_2 d}{f_1 + f_2 - d}$$

But we have $f_1 + f_2 - d = \frac{f_1 f_2}{f}$

$$\therefore \beta = -\frac{f d}{f_1}$$

CARDINAL POINTS OF A COAXIALSYSTEM OF TWO THIN LENSES:

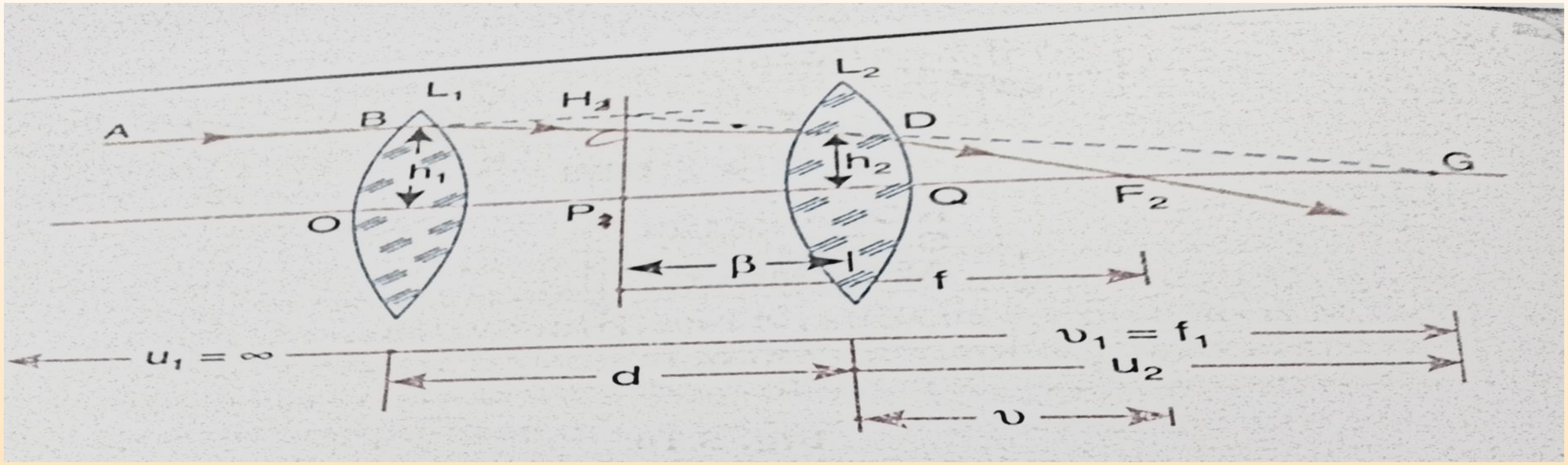


i) First principal point:

Let the distance of first principal plane $L_1P_1=\alpha$ from lens L_1 =

$$\therefore \alpha = +\frac{fd}{f_2}$$

SECOND FOCAL POINT:



The distance of second focal point F2 from the second lens L2 is

$$L_2F_2 = P_2F_2 - P_2L_2$$

$$= f - (-L_2P_2) = f + \beta$$

$$= f + \left[-\frac{fd}{f_1}\right]$$

$$L_2F_2 = f\left[1 - \frac{d}{f_1}\right]$$

Similarly the distance of first focal point F1 from first lens L1 is

$$L_1F_1 = f\left[1 - \frac{d}{f_2}\right]$$

When an optical system is located in air the position of P1 &P2 is position of Nodal points

EYEPIECE:

EYE-PIECES :

An eye-piece is a combination of lenses designed to magnify the image already formed by the objective of a telescope and microscope

EYEPIECE:

EYE-PIECES :

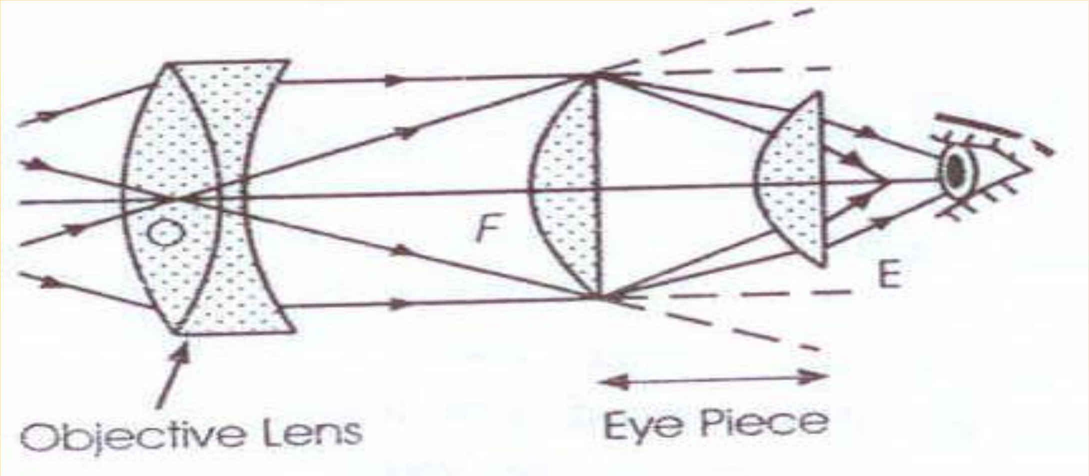
An eye-piece is a combination of lenses designed to magnify the image already formed by the objective of a telescope and microscope

The field lens has large aperture to increase the field of view.

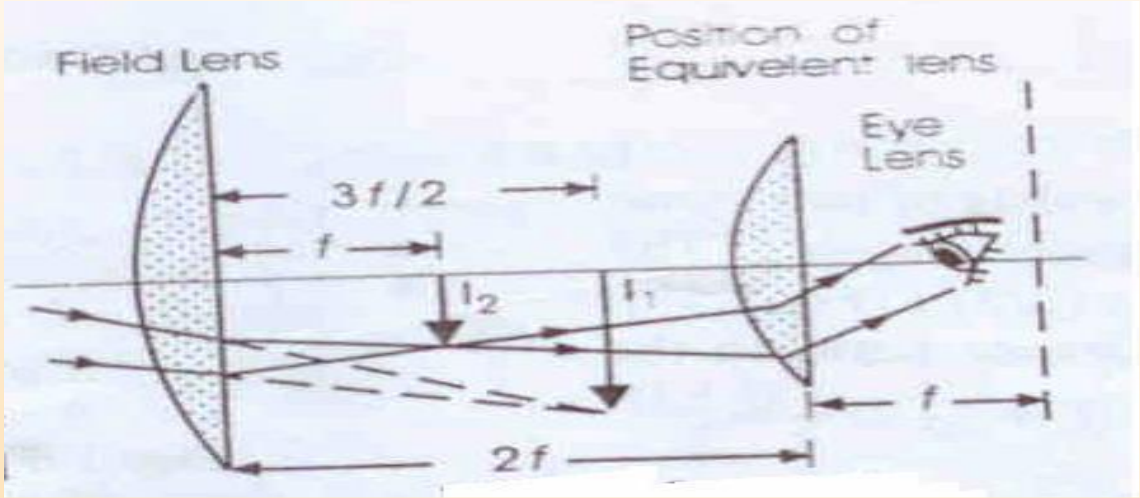
The eye lens mainly magnifies the image.

To reduce spherical aberration , the lenses taken are plano-convex.

Further the focal lengths of the two lenses and their separation are selected in such a way as to minimize the chromatic and spherical aberrations.



HUYGEN'S EYEPIECE:



Construction:

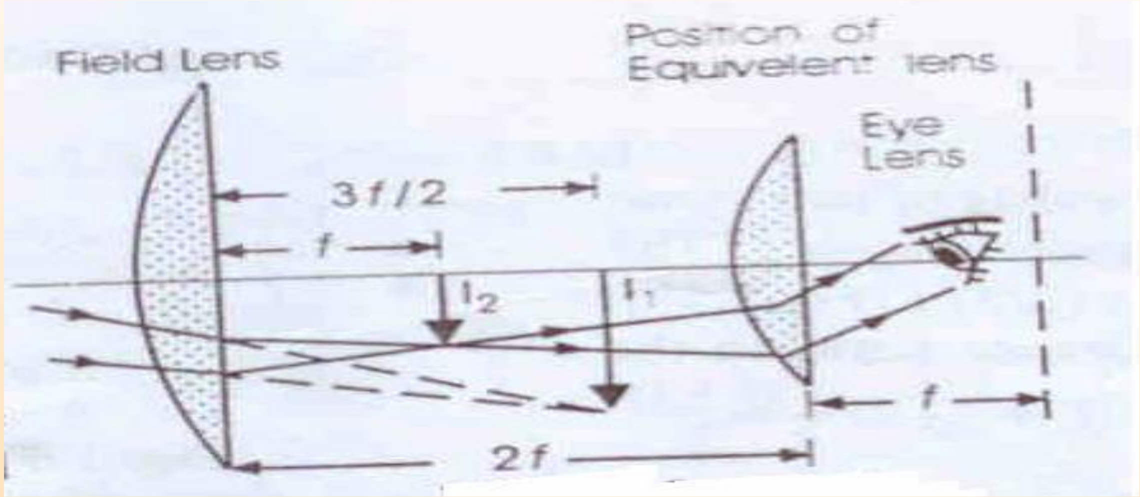
It consist of two planoconvex lens having focal length in the ratio 3:1.

Distance between two lens = difference in their focal length.

Field lens has focal length 3f and eye lens f

Planoconvex lens are placed convex surfaces towards the incident ray

HUYGEN'S EYEPIECE:



(i) The distance between the two lenses for minimum spherical aberration is given by

d = $f1 - f2$. In Huygen's eyepiece,d= 3 f - f = 2 f. Hence this eye-piece satisfies the

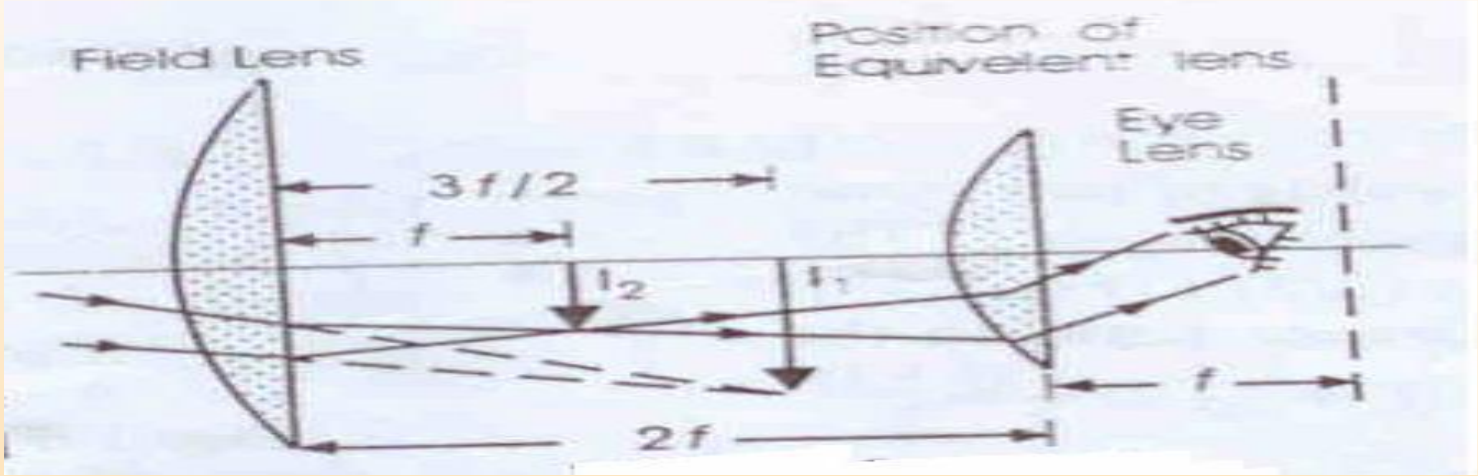
condition of minimum spherical aberration.

ii) For minimum chromatic abberation d=(f_1+f_2) /2

In Huygens eyepiece d= (3_f+f) /2 =2f

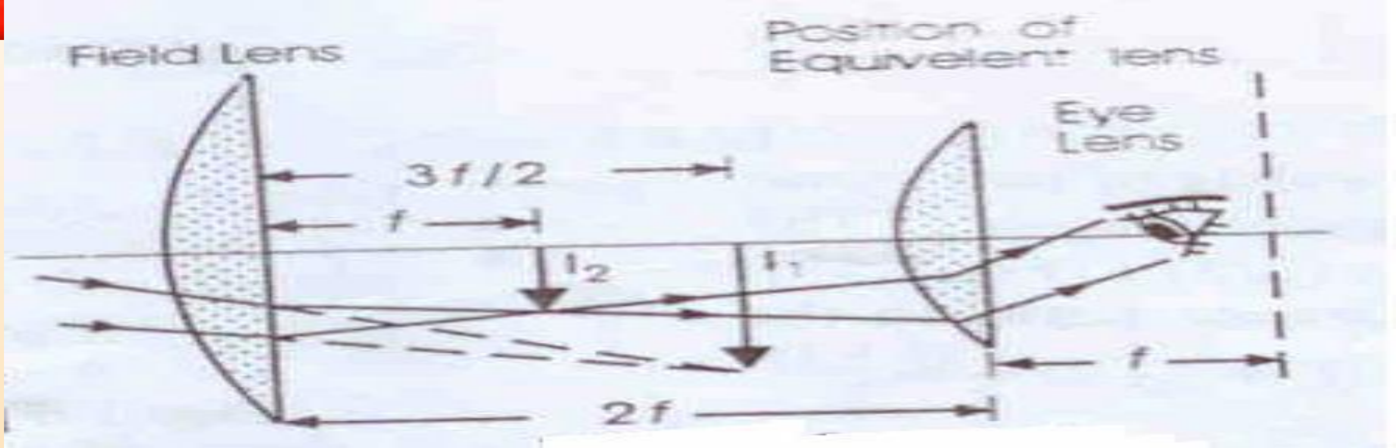
Hence this eyepiece satisfies the condition of minimum chromatic aberration.

HUYGEN'S EYEPIECE:



- **WORKING:** Objective forms an image I_1, which serves as a virtual object for field lens
- Field lens forms real inverted image I_2
- When this image is situated at principal focus of eye lens , then final image is at infinity

HUYGEN'S EYEPIECE:

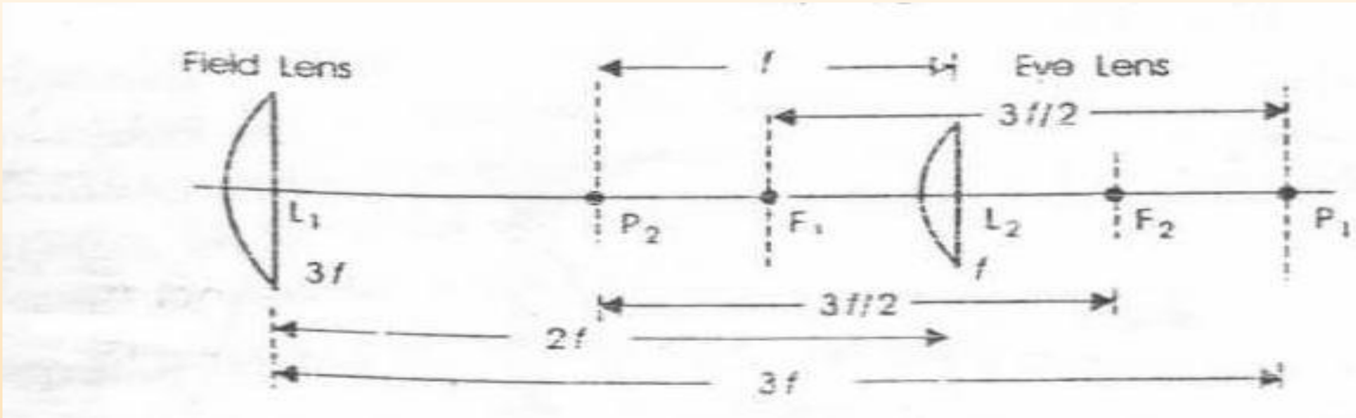


- **Equivalent focal length:**
- If F is the equivalent focal length of eyepiece , then

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{3f} - \frac{2f}{3f^2} \qquad = \frac{2}{3f} \qquad\qquad \therefore F = \frac{3f}{2}$$

CARDINAL POINTS OF HUYGENS EYEPIECE



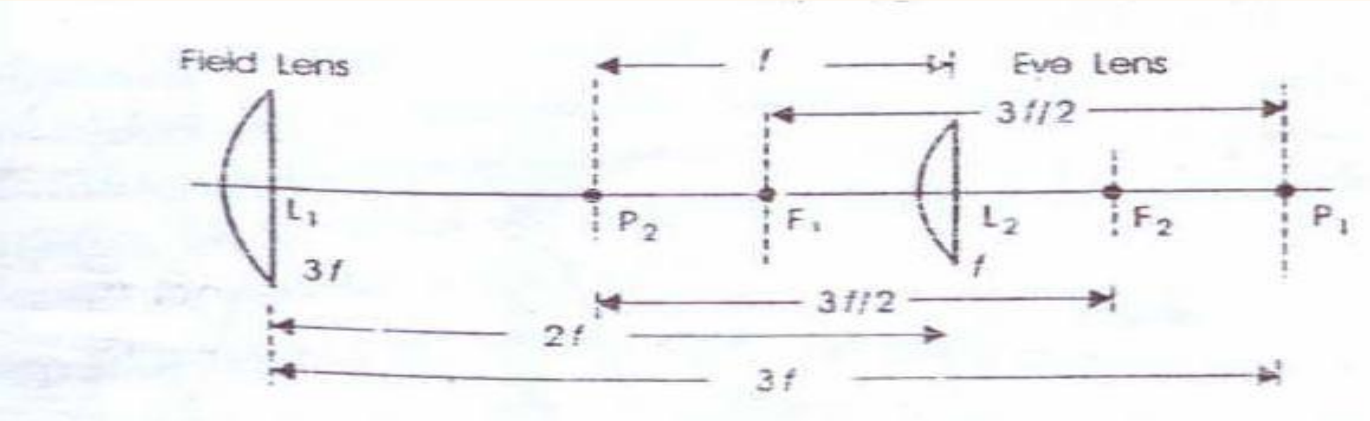
The equivalent focal length F of this eyepiece is

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \qquad \frac{1}{F} = \frac{1}{f} + \frac{1}{3f} - \frac{2f}{3f^2} \qquad \therefore F = \frac{3f}{2}$$

The second principal point is at a distance of β from eyelens is

$$\therefore \beta = -\frac{Fd}{f_1} \qquad \therefore \beta = -\frac{\frac{3}{2} f x 2f}{3f} = -f$$

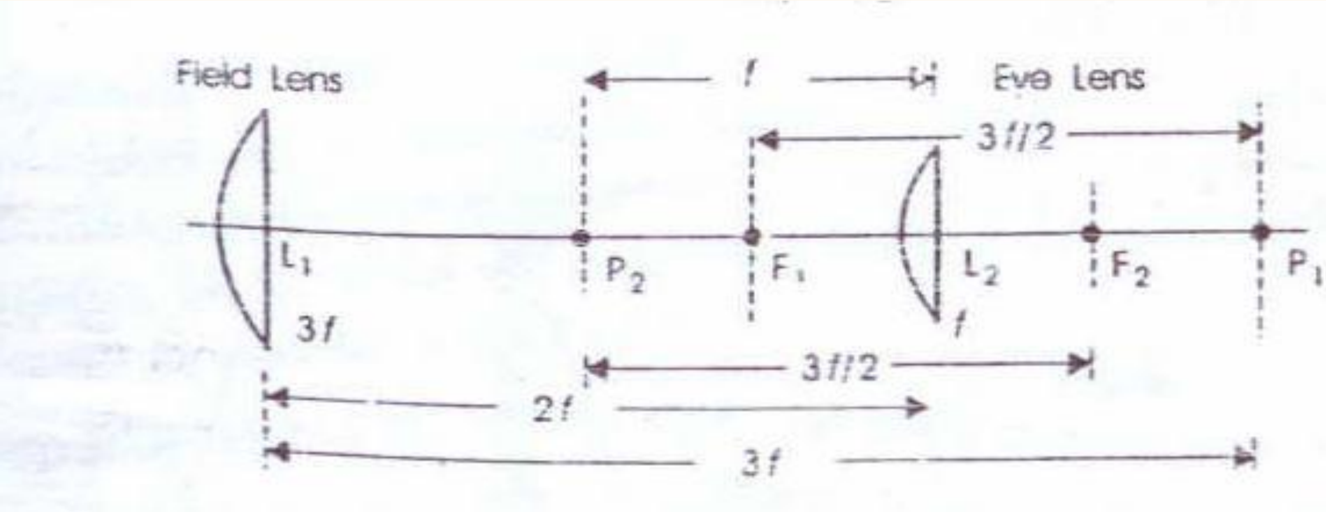
CARDINAL POINTS OF HUYGENS EYEPIECE



The first principal point at a distance α from the field lens is

$$\alpha = \frac{Fd}{f_2} \qquad \therefore \alpha = -\frac{-\frac{3}{2}fx2f}{f} = 3f$$

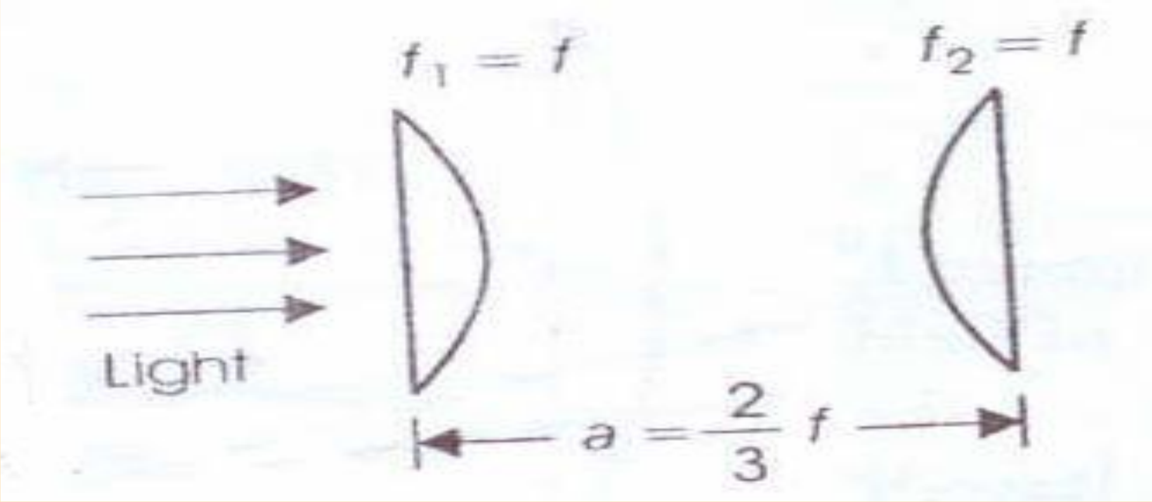
CARDINAL POINTS OF HUYGENS EYEPIECE



- Since the d distance between field lens and eyelens is 2f.
- The position of equivalent lens is 3f-2f =f i.e. It should be placed away from eyelens at P1
- P1 lies at a distance of α= 3f from field lens
- Position of focal points:
- First focal point F1 lies at a distance of 3f/2 from P1 i.e. at a distance of
- {3f/2-f} =f/2 from eyelens on side of field lens
- Second focal point F2 lies at a distance of 3f/2 from P2 i.e. at a distance of {3f/2-f} =f/2 from eyelens away from the field lens

RAMSDEN'S EYEPIECE:

CONSTRUCTION:



It consist of two planoconvex lens having focal length in the ratio 1:1

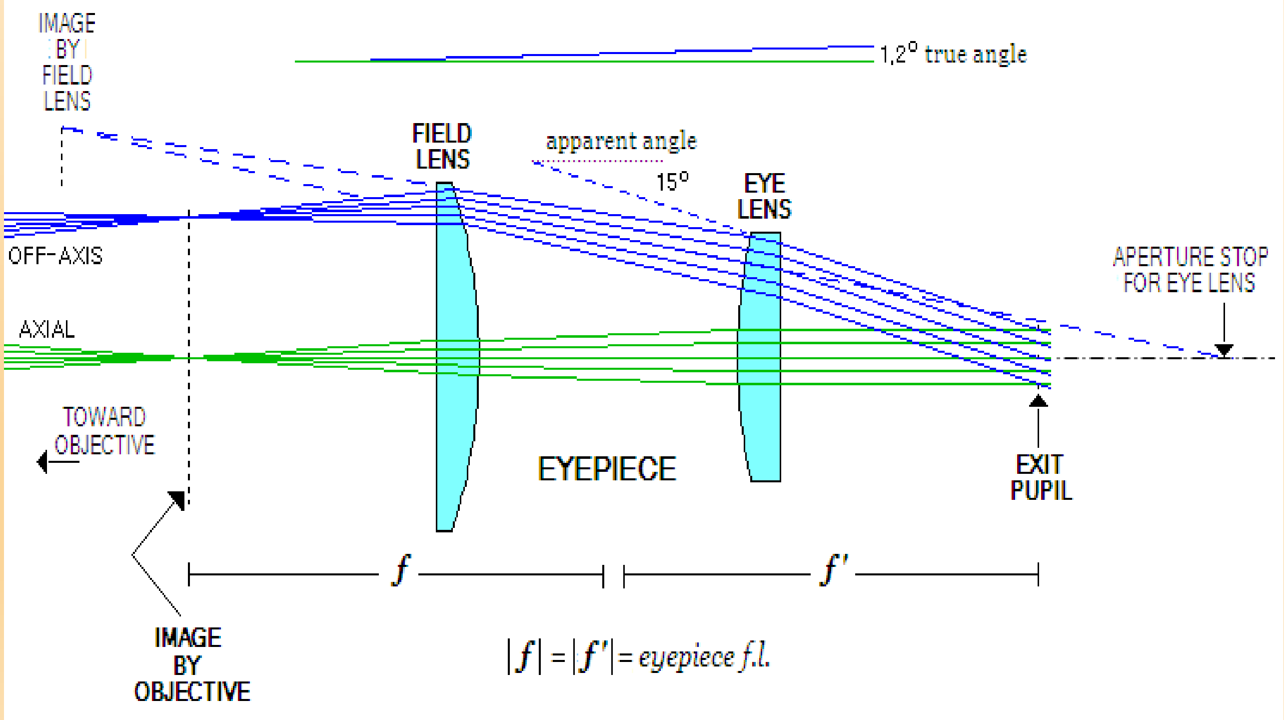
Distance between two lens is 2/3 f .

Field lens has focal length f and eye lens f

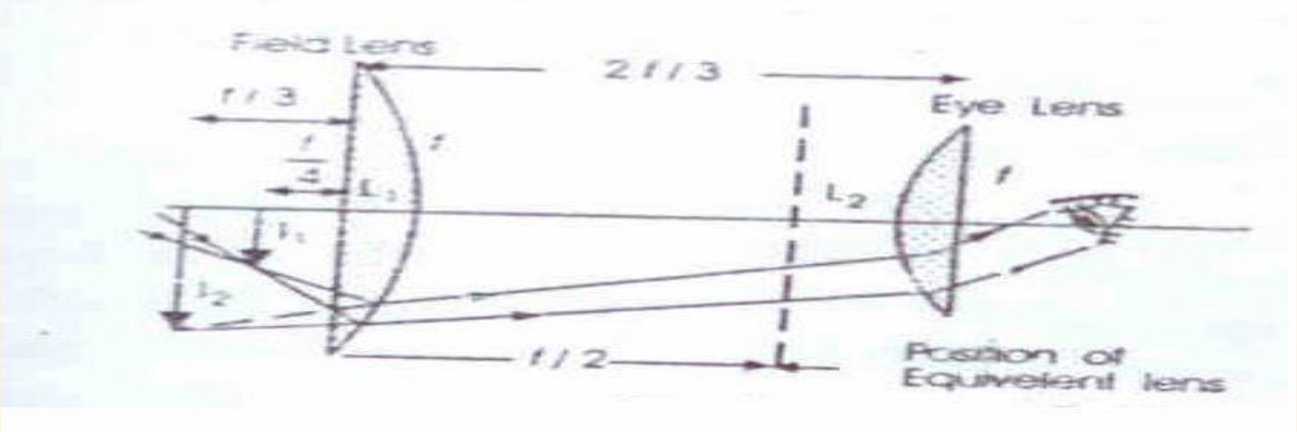
Planoconvex lens are placed convex surfaces facing each other to reduce spherical aberration

RAMSDEN'S EYEPIECE:

- For achromatism,the distance between two lens should be **$d=(f_1+f_2)/2$**
- **$=(f+f)/2=f$** But here it is **2/3 f**.Therefore This eyepiece is not free from chromatic aberration



WORKING:



- The objective forms the real inverted image I_1 of a distant object .
- This serve as an object for field lens, giving virtual image I_2.
- I_2 serve an object for eyelens,which gives the final image at infinity
- **Equivalent focal length:**
- If F is the equivalent focal length of eyepiece then

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f} - \frac{(2/3)f}{f^2}$$

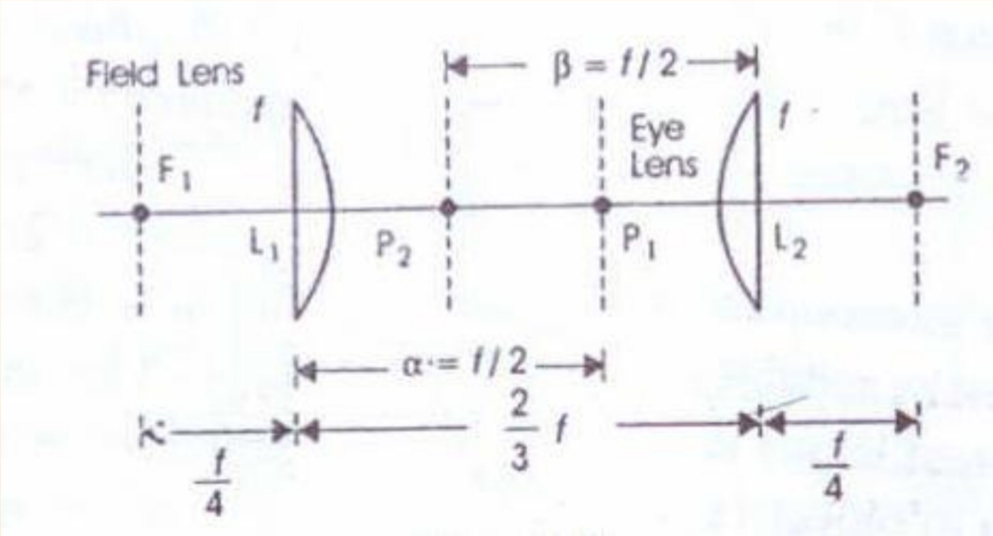
$$= \frac{2}{f} - \frac{2}{3f} = \frac{4}{3f}$$

$$\therefore F = \frac{3f}{4}$$

WORKING:

- The equivalent lens of focal length 3f/4 must be placed behind the field lens at a distance α= F d/f=[(3/4)f x(2/3)f]/f = f/2 between the field lens and eye lens
- **POSITION OF CROSS WIRE**:
- The cross wire should be at a position of **I_1.**
- Distance between Equivalent lens and image **I_1** is F=3/4 f
- i.e. since dist. Between equivalent lens and lens **L_1 =f/2** and
- between **L_1** and **I_1= f/4**
- Therefore an objective produce an image at a distance of **f/4** from lens **L_1**

CARDINAL POINTS OF RAMSDEN EYEPIECE:



- The equivalent focal length of Ramsden eyepiece F is:

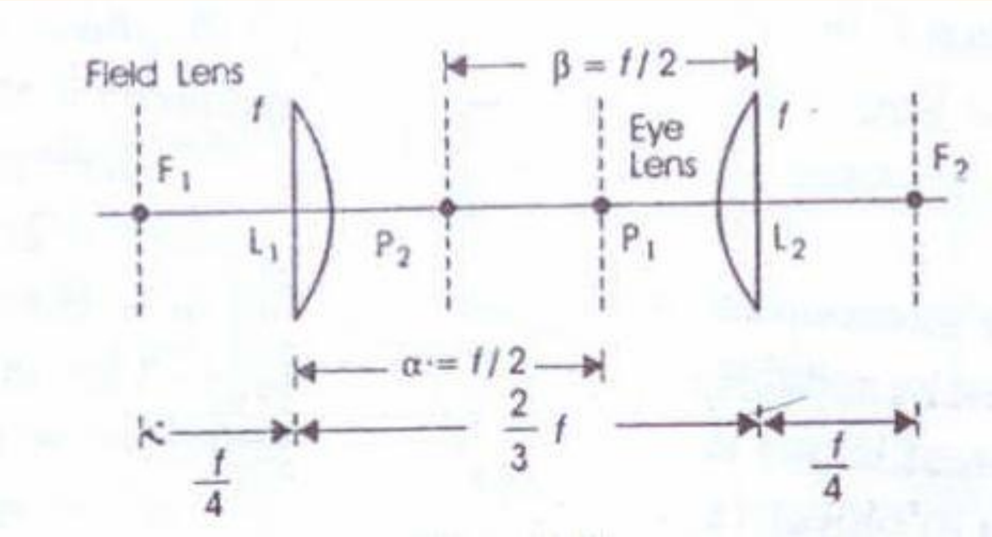
$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f} - \frac{(2/3)f}{f^2}$$

$$= \frac{2}{f} - \frac{2}{3f} = \frac{4}{3f}$$

$$\therefore F = \frac{3f}{4}$$

CARDINAL POINTS OF RAMSDEN EYEPIECE:

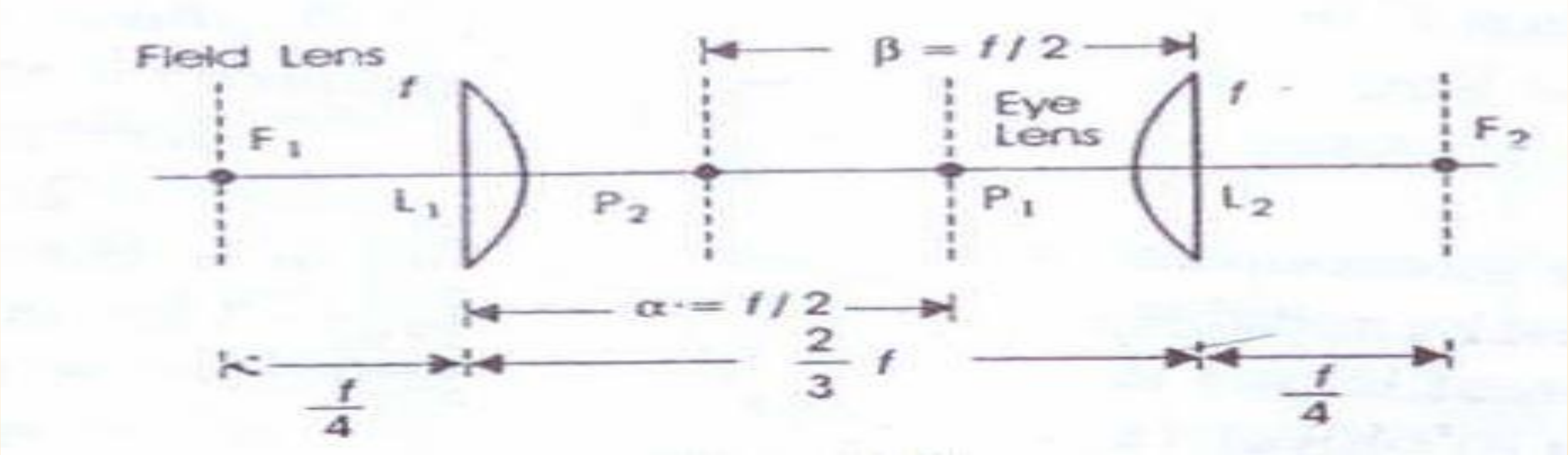


The first principal point at a distance α from the field lens is

$$\alpha = \frac{Fd}{f_2}$$

$$\alpha = \frac{\frac{3}{4}f * 2/3f}{f} = f/2$$

Thus equivalent lens lies at P_1 between L_1 & L_2 at a distance f/2 behind field lens



- The second principal point at a distance of β from eye lens L2 is

$$\therefore \beta = -\frac{Fd}{f_1}$$

$$\therefore \beta = \frac{-\frac{3}{4}f * 2/3f}{f} = -f/2$$

Position of focal points:

The first focal point F1 lies at a distance of 3f/4 from P1
i.e. at a distance of [3f/4-f/2]=f/4 from field lens L1

The second focal point F2 lies at a distance of 3f/4 from P2
i.e. at a distance of [3f/4-f/2]=f/4 behind eyelens away from field lens lens

COMPARISION

Huygen,s eyepiece

- The image of the object formed by the objective falls in between the two lenses. Therefore, no cross wires can be used. For this reason. it is called a negative eyepiece.
- It satisfies the condition for minimum spherical aberration.
- It satisfies the condition for
- achromatism.
- It is generally used for biological
- observations where no measurements are required.

Ramsdan's Eyepiece

- The image of the object formed by the objective lies in front of the field lens. Therefore, cross wires can be used. For this reason, it is called a positive

  eyepiece.
- It does not satisfy the condition for minimum spherical aberration.
- It does not satisfy the condition for achromatism.
- It is used with instruments meant for
- physical measurements.