UNIT –II

STATIONARY WAVES

FOR B.Sc. S.Y.

BY

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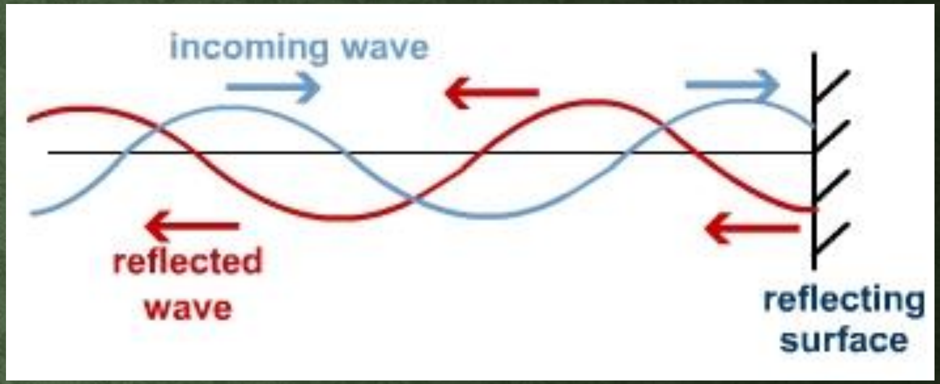
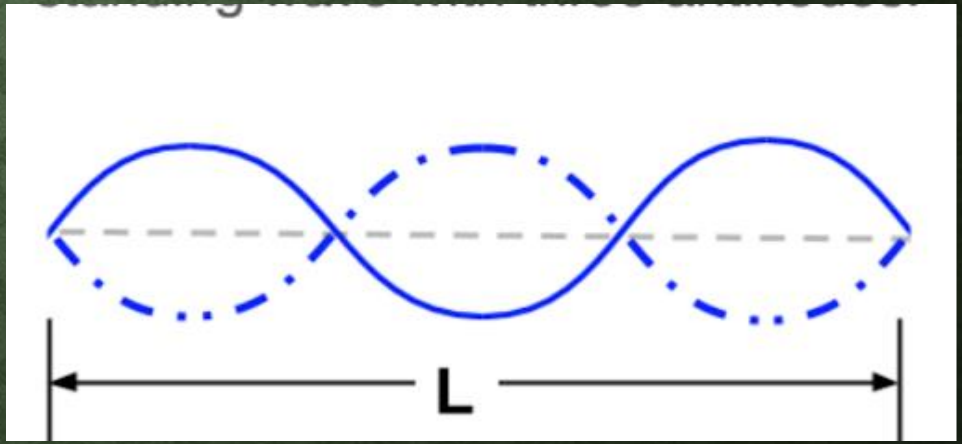
STATIONARY WAVES:

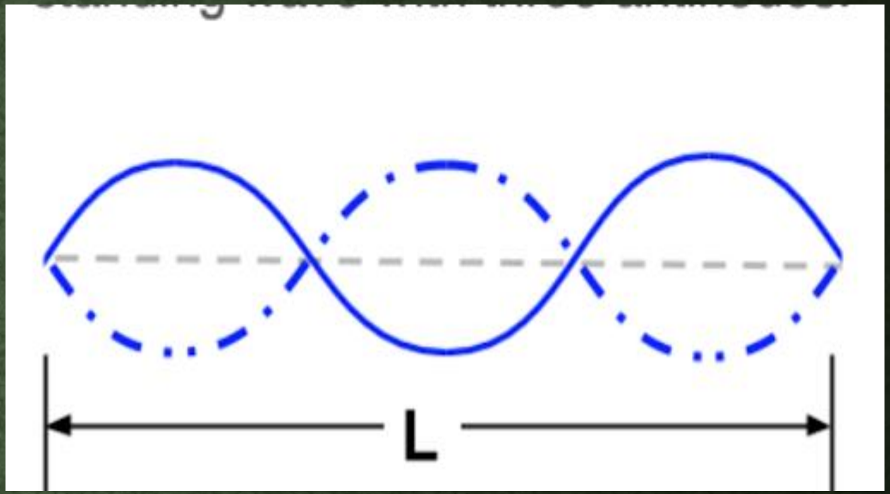
Formation of Stationary waves:

When two simple harmonic progressive waves of same amplitude, frequency and wavelength are travelling along the same path but exactly in opposite direction when superpose with each other, the resultant formed wave is in the form of loops called stationary waves





CHARACTERISTICS OF STATIONARY WAVES:



1. Nodes and antinodes are formed
2. Distance between two successive nodes or antinodes $=\lambda/2$
3. All particles except at nodes, vibrates SHM
4. Amplitude gradually increases from node to antinode
5. All particles in same loop are in phase
6. Does not travels
7. Does not transfer energy

ANALYTICAL TREATMENT
OPEN END ORGAN PIPE OR STRING FREE AT OTHER END

Consider SHPW (Incident waves)

$$y_1 = a \sin\frac{2\pi}{\lambda}(vt - x) \text{ ---- (1)}$$

Reflected wave

$$y_2 = a \sin\frac{2\pi}{\lambda}(vt + x) \text{ ---- (2)}$$

The resultant displacement of the wave

$$y = y_1 + y_2$$

$$y = a \sin\frac{2\pi}{\lambda}(vt - x) + a \sin\frac{2\pi}{\lambda}(vt + x) \text{ ---(3)}$$

ANALYTICAL TREATMENT

OPEN END ORGAN PIPE OR STRING FREE AT OTHER END

$$y = 2a\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi vt}{\lambda} \text{ ------(4)}$$

The resultant vibration of a particle represents SHM

Amplitude of resultant vibration is

$$A = 2a\cos\frac{2\pi x}{\lambda} \text{ ------} \qquad (5)$$

The velocity of particle at any instant of time is $\frac{dy}{dt}$

Differentiating equation (4) wrt. Time

$$\frac{dy}{dt} = \frac{4\pi av}{\lambda} cos\frac{2\pi x}{\lambda} cos\frac{2\pi vt}{\lambda} \qquad (6)$$

ANALYTICAL TREATMENT

OPEN END ORGAN PIPE OR STRING FREE AT OTHER END

Acceleration of the particle at any instant is $\frac{d^2y}{dt^2}$

Differentiating equation(6) with time

$$\frac{d^2y}{dt^2} = -\frac{8\pi^2 a v^2}{\lambda^2} \cos\frac{2\pi x}{\lambda} \sin\frac{2\pi v t}{\lambda} \qquad (7)$$

strain or compression at any point is $\frac{dy}{dx}$

Differentiating equation(4) with x

$$\frac{dy}{dx} = -\frac{4\pi a}{\lambda} \sin\frac{2\pi x}{\lambda} \sin\frac{2\pi v t}{\lambda} --- \qquad (8)$$

Eq. shows amplitude, velocity, acceleration and strain changes with time and positions

ANALYTICAL TREATMENT

(1) CHANGES WITH RESPECT TO POSITION:

Consider the positions, where

$sin\frac{2\pi x}{\lambda} = 0$ and $\cos\frac{2\pi x}{\lambda} = \pm 1$

From eq, (4) to (8)

Displacement, $y = \pm 2a\, sin\frac{2\pi vt}{\lambda}$-------(9)

Amplitude, A= $\pm 2a$ ----------------(10)

Velocity, $\frac{dy}{dt} = \pm\frac{4\pi av}{\lambda} cos\frac{2\pi vt}{\lambda}$ ---(11)

ANALYTICAL TREATMENT

(1) CHANGES WITH RESPECT TO POSITION:

Acceleration, $\frac{d^2y}{dt^2} = \pm \frac{8\pi^2 a v^2}{\lambda^2} sin \frac{2\pi vt}{\lambda}$ ---(12)

Strain, $\frac{dy}{dx} = 0$ ----------------------(13)

As strain is zero, these positions corresponds to Antinodes

$$\therefore \ sin \frac{2\pi x}{\lambda} = 0$$

But sin mπ = 0

Where m=0,1,2,3 etc

ANALYTICAL TREATMENT

(1) CHANGES WITH RESPECT TO POSITION:

$$\therefore \; sin\frac{2\pi x}{\lambda} = 0$$

But sin mπ = 0 Where m=0,1,2,3 etc

$$\therefore \; sin\frac{2\pi x}{\lambda} = \text{m}\pi$$

OR

$x = \frac{m\lambda}{2}$ or x= $0, \frac{\lambda}{2}, \lambda , \frac{3\lambda}{2}$ ---- etc

Thus antinodes are equidistant and separated by $\frac{\lambda}{2}$

ANALYTICAL TREATMENT
(1) CHANGES WITH RESPECT TO POSITION:

$At\ x = 0$, the position of interface is an antinode

Position of open end of organ pipe – Antinode

Free end of stretched string - Antinode

ANALYTICAL TREATMENT

(2) CHANGES WITH RESPECT TO POSITION:

Consider the positions, where

$sin\frac{2\pi x}{\lambda} = \pm 1$ and $\cos\frac{2\pi x}{\lambda} = 0$

From eq, (4) to (8)

Displacement, y = 0-------(14)

Amplitude, A= 0 -------(15)

Velocity, $\frac{dy}{dt} = 0$ ------ (16)

ANALYTICAL TREATMENT
(2) CHANGES WITH RESPECT TO POSITION:

Acceleration, $\frac{d^2y}{dt^2}$ =0 ---------(17)

Strain, $\frac{dy}{dx} = \pm \frac{4\pi a}{\lambda} \sin \frac{2\pi vt}{\lambda}$ --------(18)

As strain is maximum, these positions corresponds to nodes

$$\therefore \ cos \frac{2\pi x}{\lambda} = 0$$

But $cos(2\,m + 1)\frac{\pi}{2} = 0$ Where m=0,1,2,3 etc

ANALYTICAL TREATMENT

(2) CHANGES WITH RESPECT TO POSITION:

$$\therefore \; cos\frac{2\pi x}{\lambda} = cos(2\,m+1)\frac{\pi}{2} = 0$$

$$\therefore \frac{2\pi x}{\lambda} = (2\,m+1)\frac{\pi}{2}$$

OR

$x = \frac{(2\,m+1)\lambda}{4}$ or x= $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$ ---- etc

Thus nodes are equidistant and separated by $\frac{\lambda}{2}$

ANALYTICAL TREATMENT

(1) CHANGES WITH RESPECT TO TIME:

Consider the instant of time, where

$sin\frac{2\pi vt}{\lambda} = 0$ and $\cos\frac{2\pi vt}{\lambda} = \pm 1$

From eq, (4) to (8)

Displacement, $\mathrm{y} = 0$

Amplitude, A= $2a\ cos\frac{2\pi x}{\lambda}$ (Independent of time)

Velocity, $\frac{dy}{dt} = \pm\frac{4\pi av}{\lambda}cos\frac{2\pi vt}{\lambda}$

ANALYTICAL TREATMENT
(1) CHANGES WITH RESPECT TO TIME:

Acceleration, $\frac{d^2y}{dt^2}=0$

Strain, $\frac{dy}{dx}=0$

$$\therefore \; sin\frac{2\pi vt}{\lambda} = 0$$

But sin mπ = 0

Where m=0,1,2,3 etc

ANALYTICAL TREATMENT
(1) CHANGES WITH RESPECT TO TIME:

$$\therefore \sin\frac{2\pi vt}{\lambda} = m\pi$$

OR

$$t = \frac{m\lambda}{2v}$$

But $\frac{v}{\lambda} = n = \frac{1}{T}$

$$t = \frac{mT}{2}$$

$$t = 0, \frac{T}{2}, T, \frac{3T}{2} \text{ ---- etc}$$

Twice in each time period , particle pass through their mean position

ANALYTICAL TREATMENT
(2) CHANGES WITH RESPECT TO TIME:

Consider the instant of time, where

$sin\frac{2\pi vt}{\lambda} = \pm 1$ and $\cos\frac{2\pi vt}{\lambda} = 0$

From eq, (4) to (8)

Displacement, $y = \pm 2a\ cos\frac{2\pi x}{\lambda}$

Amplitude, A= $2a\ cos\frac{2\pi x}{\lambda}$ (Independent of time)

Velocity, $\frac{dy}{dt} = 0$

ANALYTICAL TREATMENT
(1) CHANGES WITH RESPECT TO TIME:

Acceleration, $$\frac{d^2y}{dt^2} = \pm \frac{8\pi^2 a v^2}{\lambda^2} \cos\frac{2\pi x}{\lambda}$$

Strain, $$\frac{dy}{dx} = \mp \frac{4\pi a}{\lambda} \sin\frac{2\pi x}{\lambda}$$

$$\therefore \sin\frac{2\pi v t}{\lambda} = \pm 1$$

But $$\sin(2m+1)\frac{\pi}{2} = \pm 1$$

Where m=0,1,2,3 etc

ANALYTICAL TREATMENT
(2) CHANGES WITH RESPECT TO TIME:

$$\therefore \; sin\frac{2\pi vt}{\lambda} = sin(2\,m + 1)\frac{\pi}{2} = \pm\,1$$

$$\therefore \frac{2\pi vt}{\lambda} = (2\,m + 1)\frac{\pi}{2}$$

$t = \frac{(2\,m+1)\lambda}{4v}$ But $\frac{v}{\lambda} = n = \frac{1}{T}$

t= $\frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}$ ---- etc

Thus at these instants particle is at extreme position called stationary instants.