UNIT-III
FREE AND FORCED VIBRATIONS:

By

Bhanudas Narwade

Degloor college,Degloor

FREE VIBRATIONS:

FREE VIBRATIONS: When body is displaced from its mean position and released, body vibrates with its own natural frequency

Time period of body executing SHM depends on

1) dimensions

2) elastic constants

If no resistance is applied, ,amplitude remains constant

Period of free vibration is called free period

UNDAMPED VIBRATIONS:

UNDAMPED VIBRATIONS: Amplitude of vibrations remains constant throughout its motion

No loss of energy

For SHM ,KE for displacement y is $\frac{1}{2}m\left(\frac{dy}{dt}\right)^2$

PE of particle $\frac{1}{2}K\ y^2$

Total energy at any instant

$= \frac{1}{2}m\left(\frac{dy}{dt}\right)^2 + \frac{1}{2}K\ y^2$

UNDAMPED VIBRATIONS:

For undamped vibrations no loss of energy

$\therefore \frac{1}{2} m \left(\frac{dy}{dt}\right)^2 + \frac{1}{2} K\, y^2 =$ constant --------(1)

Differentiating wrt t

$m \frac{d^2y}{dt^2}$ +Ky=0 -----------(2)

Or

$\frac{d^2y}{dt^2} + \left(\frac{K}{m}\right) y = 0$ -------------(3)

Eq. 3 is similar to

$\frac{d^2y}{dt^2} + \omega^2\, y = 0$ ----------(4) Here $\omega^2 = \left(\frac{K}{m}\right)$

UNDAMPED VIBRATIONS:

Solution of equation is

$$y = a\,Sin\,(\omega t - \alpha)$$

$$y = a\,Sin\,\left[\sqrt{\frac{K}{m}}t - \alpha\right]$$

Frequency of oscillation is

$$n = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

This is ideal case

DAMPED VIBRATIONS:

Free Damped vibrations : When body vibrates in air medium , frictional forces and constantly energy is dissipated.

Amplitude of swing gradually decreases with time finally oscillations die out

Dissipated energy appears as a heat either within system or surroundings

Dissipative force due to friction ∝ velocity of particle at that instant

$\mu \frac{dy}{dt}$ dissipative force due to friction

DAMPED VIBRATIONS:

The differential equation in the case of free damped vibrations

$$m\frac{d^2y}{dt^2} + Ky + \mu\frac{dy}{dt} = 0 \quad \text{-------(1)}$$

Or

$$\frac{d^2y}{dt^2} + \left(\frac{\mu}{m}\right)\frac{dy}{dt} + \left(\frac{K}{m}\right)y = 0 \quad \text{---(2)}$$

This equation is similar to general differential eq.

$$\frac{d^2y}{dt^2} + 2b\frac{dy}{dt} + K^2y = 0 \quad \text{-----(3)}$$

DAMPED VIBRATIONS:

The solution of this equation is

$$y = ae^{-bt}\ (\omega t - \alpha)$$

General solution

$$y = Ae^{\left(-b+\sqrt{b^2-k^2}\right)t} + B\ e^{\left(-b-\sqrt{b^2-k^2}\right)t}$$

Here b $= \frac{\mu}{2m}$ and k^2=K/m

And

$$\omega = \sqrt{k^2 - b^2}$$

DAMPED VIBRATIONS:

$$\omega = \sqrt{\frac{K}{m} - \frac{\mu^2}{4m^2}} \text{ -------(4)}$$

Frequency

$$n = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{k^2 - b^2}$$

Period

$$T = \frac{1}{n} = 2\pi\frac{1}{\sqrt{k^2 - b^2}}$$

FORCED VIBRATIONS:

Forced Vibration: External periodic force constantly applied
Continues to oscillate under the influence
Initially amplitude increases and then decreases becomes minimum and then again increases
Finally body will forced to vibrate with same frequency
Driven frequency and natural frequency are different
Amplitude of vibration depends upon difference of both freq.
If difference is small ,amplitude is large

FORCED VIBRATIONS:

Differential equation:

$m\frac{d^2y}{dt^2}$ +Ky+μ $\frac{dy}{dt}$=Fsin pt ---------(1)

P is angular ferq. Of applied force

Solution of equation (1) is

$y = a\ Sin\ (pt - \alpha)$ ------(2)

$\frac{dy}{dt} = ap\ cos\ (pt - \alpha)$----(3)

$\frac{d^2y}{dt^2} = -\mathrm{a}p^2\ \sin(pt - \alpha) = -p^2\mathrm{y}$----(4)

FORCED VIBRATIONS:

Substituting in eq. I

$-\mathrm{map}^2 \sin(pt-\alpha) + \mathrm{K}a\, Sin\,(pt-\alpha) + \mu ap\, cos\,(pt-\alpha) = \mathrm{Fsin\,pt}$

$-\mathrm{mp}^2 a[\sin pt\, cos\alpha - \cos pt\ \sin\alpha]$

$+\ \mathrm{Ka}\ [\sin pt\, cos\alpha - \cos pt\ \sin\alpha]$

$+\ \mu ap[\cos pt\, cos\alpha + sinpt\, sin\alpha] - \mathrm{Fsin\ pt} = 0$ ----(5)

$-\mathrm{mp}^2 a\, cos\alpha + Ka\cos\alpha + \mu ap\, sin\alpha - F = 0$ --------(6)

$+\ \mathrm{mp}^2 a\, sin\alpha - Ka\sin\alpha + \mu ap\, cos\alpha = 0$ --------------(7)

Dividing eq. 7 by cosα and simplifying

$tan\alpha = \frac{\mu p}{K-mp^2} = \frac{A}{B}$------(8)

FORCED VIBRATIONS:

$$tan\alpha = \frac{\mu p}{K - mp^2} = \frac{A}{B} \text{------}(8)$$

from eq.8

$$sin\alpha = \frac{A}{\sqrt{A^2+B^2}} \text{-------}(9)$$

$$\cos\alpha = \frac{B}{\sqrt{A^2+B^2}} \text{ -----}(10)$$

Dividing eq.6 by cos α

$$-mp^2a + Ka + \mu ap\ tan\alpha - \frac{F}{cos\alpha} = 0\text{----}(11)$$

Or

$$a[(K - mp^2) + \mu p\ tan\alpha] = \frac{F}{cos\alpha}$$

FORCED VIBRATIONS:

But $K - mp^2 = B$ and $\mu p = A$

Substituting for $\tan\alpha$ and $\cos\alpha$

$$a\left[B + \frac{A^2}{B}\right] = \frac{F\sqrt{A^2+B^2}}{B}\text{-----}$$

$$a = \frac{F}{\sqrt{A^2+B^2}}\text{----}$$

Substituting values of A and B

$$a = \frac{F}{\sqrt{\mu^2p^2+(K-mp^2)^2}}\text{--(12)}$$

$$y = a\sin(pt - \alpha)\text{------}$$

$$y = \frac{F}{\sqrt{\mu^2p^2+(K-mp^2)^2}}\sin(pt - \alpha)$$

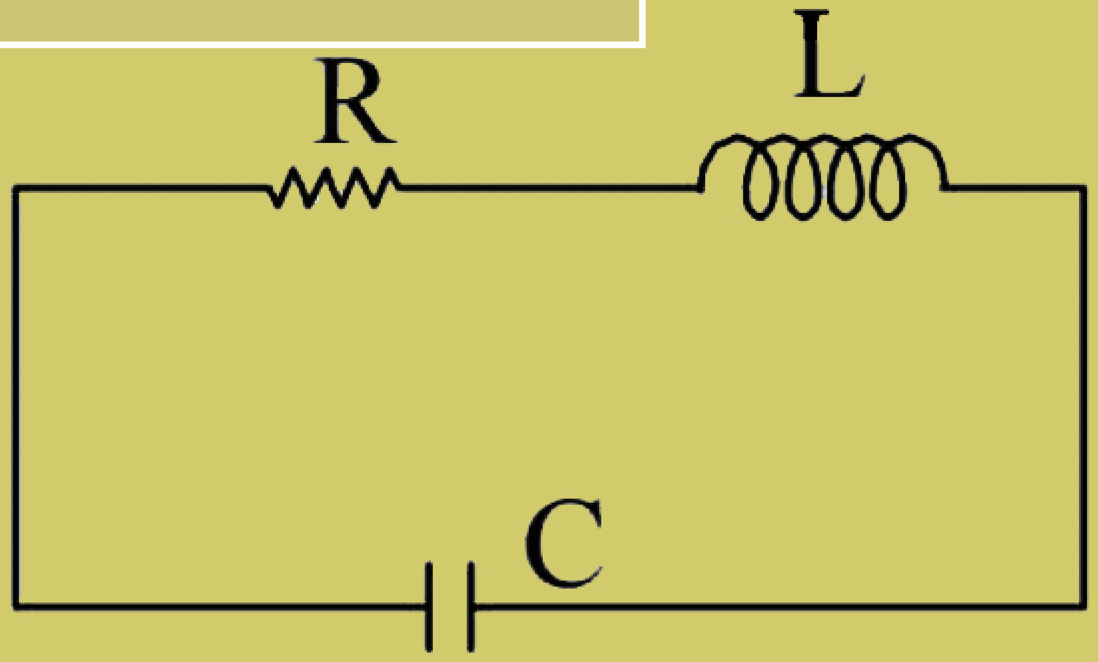
FORCED VIBRATIONS:

$$y = \frac{F}{\sqrt{\mu^2 p^2 + (K - mp^2)^2}} \sin(pt - \alpha)$$

General solution will include both solution for free and forced vibrations

$$y = ae^{-bt}\ (\omega t - \alpha) + \frac{F}{\sqrt{\mu^2 p^2 + (K - mp^2)^2}} \sin(pt - \alpha)$$

DAMPED SHM IN ELECTRICAL CIRCUIT:



LCR: Electrical circuit containing R ,L and C

When C is charged :q_0 be charge on it and during discharge looses charge through R

$\frac{dI}{dt}$=Rate of change of current

q is charge on C,

PD across condenser=$\frac{q}{C}$

DAMPED SHM IN ELECTRICAL CIRCUIT:

PD across R=RI

Back emf in L=-$L\frac{dI}{dt}$

$\therefore \frac{q}{C} + RI = -L\frac{dI}{dt}$------(1)

$\frac{q}{C} + RI + L\frac{dI}{dt} = 0$----(2)

But I= $\frac{dq}{dt}$ and $\frac{dI}{dt} = \frac{d^2q}{dt^2}$

$L\frac{d^2q}{dt^2}$+R $\frac{dq}{dt}$+ $\frac{q}{C}$

DAMPED SHM IN ELECTRICAL CIRCUIT:

$$= L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0$$

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q = 0$$

Put $\frac{R}{L} = 2b$ and $\frac{1}{LC} = K^2$

$$\frac{d^2q}{dt^2} + 2b\frac{dq}{dt} + K^2q = 0 \text{-----(3)}$$

DAMPED SHM IN ELECTRICAL CIRCUIT:

The frequency $n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$

When R is negligible, then $\frac{R^2}{4L^2}$ =0

$n = \frac{1}{2\pi\sqrt{LC}}$---(4)

RESONANCE AND SHARPNESS OF RESONANCE:

In forced vibration

$$y = ae^{-bt}(\omega t - \alpha) + \frac{F}{\sqrt{\mu^2 p^2 + (K - mp^2)^2}} \sin(pt - \alpha)$$

If viscosity of medium is small. Amplitude is

$$\frac{F}{\sqrt{\mu^2 p^2 + (K - mp^2)^2}}$$

Driving force is maximum when $K - mp^2 = 0$ or $K = mp^2$

Or $\quad p = \sqrt{\frac{K}{m}}$

RESONANCE AND SHARPNESS OF RESONANCE:

Amplitude is infinite if μ is zero

Oscillations will have maximum amplitude

State of vibrations of system is called Resonance

When forced frequency equals to natural frequency,

Resonance takes place

Amplitude at resonance$=\frac{F}{\mu p} = \frac{F}{\mu\sqrt{K/m}}$

OR Amplitude at resonance$=\frac{F}{\mu}\sqrt{\frac{m}{K}}$

SHARPNESS OF RESONANCE:

Sharpness of Resonance: Fall in amplitude with change in frequency on each side of maximum amplitude

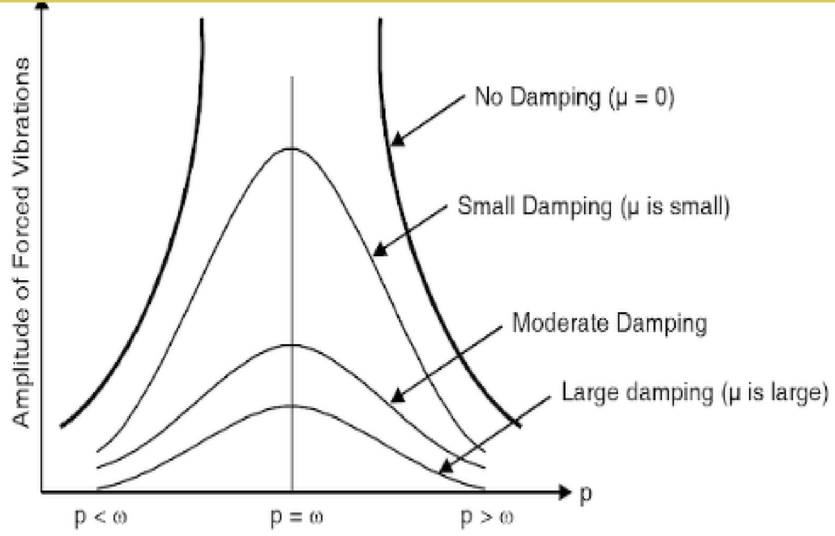


Fig. 4.5

Sharpness of Resonance : *Sharpness of resonance refers to the fall in amplitude with change in frequency on each side of the maximum amplitude.* The resonance is said to be sharp if the amplitude falls off rapidly with the deviation of ω from *p*. The degree of sharpness depends upon the magnitude of damping. For sharpness of resonance, μ (damping coefficient) must be small. In the sonometer experiment, we get sharp resonance due to the small damping coefficient of the wire.

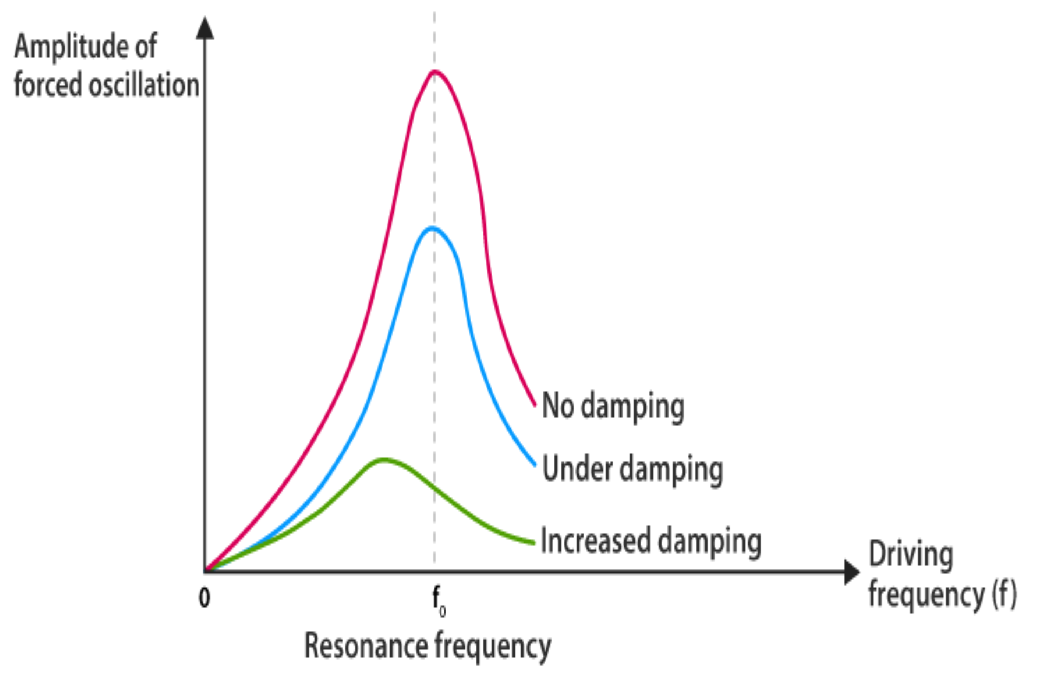
SHARPNESS OF RESONANCE:

Sharpness of Resonance: Fall in amplitude with change in frequency on each side of maximum amplitude

Kinetic energy per unit force is called response R

$$R \propto \frac{1}{\mu}$$

SHARPNESS OF RESONANCE:



1.When frictional forces are Absent (μ=0), R is infinite
And sharpness of resonance is Maximum

2. R decreases with increase in μ

3.Sharpness dies with small change in p/ω

In resonance tube exp. damping force is large as Blue curve

In sonometer wire damping force is small as Red curve

PHASE OF RESONANCE:

Phase of forced vibration and driven force are related to

$$tan\alpha = \frac{\mu p}{K - mp^2}$$

$$tan\alpha = \frac{\frac{\mu p}{m}}{(\frac{K}{m} - p^2)}$$

At resonance $\frac{K}{m} = p^2$ and tanα=infinity

It means for p/ω=1, α =π/2

For p/ω=> 1, α > π/2

For p/ω=< 1, α < π/2

EXAMPLES OF FORCED AND RESONANT VIBRATION:

1) Marching of soldiers on bridge

2) Pushing of swing

3) Prongs of tunning fork kept on table top

MCQ

1. If pendulum is displaced in vacuum, its amplitude of oscillation:
    a) Gradually decreases with time
    b) Remains constant
    c) Gradually increases with time
    d) Initially increases then decreases

MCQ:

MCQ

2. **When body vibrating freely has no resistance offered to its motion, its amplitude…**

   a) Increases with time

   b) Decreases with time

   c) Remains constant

   d) Initially increases then decreases

MCQ:

3. Period of free undamped vibration is :

a) $T = \pi\sqrt{\frac{m}{K}}$

b) $T = \frac{1}{2\pi}\sqrt{\frac{m}{K}}$

c) $T = 2\pi\sqrt{\frac{m}{K}}$

d) $T = \frac{1}{\pi}\sqrt{\frac{m}{K}}$

MCQ:

4. Frequency of free undamped vibration is :

a) n= $\pi\sqrt{\frac{m}{K}}$

b) n= $\frac{1}{2\pi}\sqrt{\frac{m}{K}}$

c) n= $2\pi\sqrt{\frac{m}{K}}$

d) n= $\frac{1}{\pi}\sqrt{\frac{m}{K}}$

MCQ:

5. The period (T)of undamped oscillations is :

a) $T = \frac{2\pi}{n}$

b) $T = \frac{\pi}{n}$

c) $T = 2\pi n$

d) $T = \pi n$

MCQ:

6. When a body is maintained in a state of vibration by a periodic force, the type of vibration is:

   a) Forced vibration

   b) Free damped vibration

   c) Free undamped vibration

   d) None of above correct

MCQ:

7. The existence of damping can..
    a) Decreases in amplitude
    b) Increases in amplitude
    c) Maintain constant in amplitude
    d) None of above are correct

MCQ:

8 At resonance amplitude of oscillation is

a) Zero

b) Minimum

c) In between zero and maximum

d) Maximum