UNIT-III

TRANSPORT PHENOMENON IN GASES

BY

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TRANSPORT PHENOMENON IN GASES

Kinetic Theory: Random motion, Molecular collision

Molecules possesses mass , momentum and energy

If not in equilibrium state or steady state , transportation occurs

Transport Phenomenon:

Phenomenon explained on the basis of movement of gas molecules to bring to steady state.

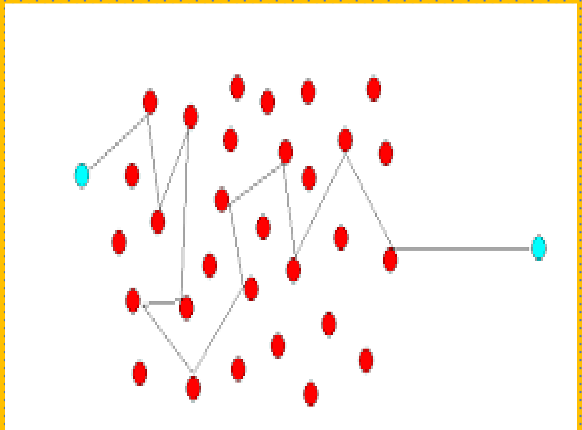
Viscosity : Transport of Momentum

Thermal conductivity : Transport of thermal energy

Self diffusion : Transport of mass

Mean Free Path:

Free Path : Distance covered by a gas molecule between any two successive collision



Mean free Path(λ): Average distance covered by a gas molecule between any two successive collision

If S is total distance travelled after N collisions

Mean free path $\lambda = S/N$

Expression for mean free path:

For simplification Consider

i) Only the molecule under consideration is in motion and all other are at rest.

ii) The sphere of influence of has diameter "d"

v is the average velocity of molecule A

n is no. of molecules per cc

In one second **A** will collides with all molecules, the centre of which lies within radius d of cylinder and length **v**

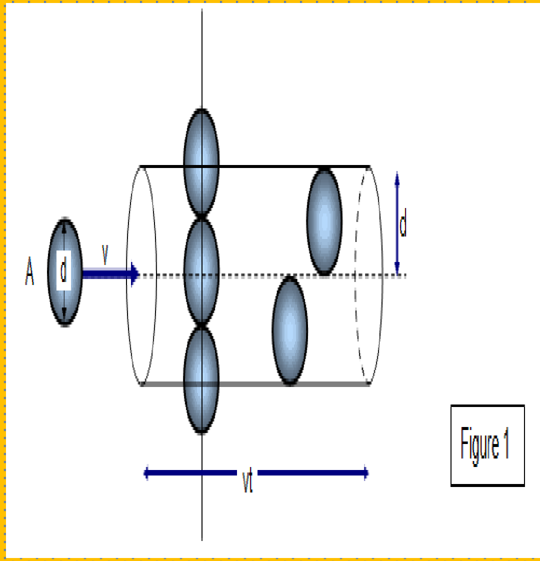
Volume of cylinder= $\pi d^2 v$

No. of molecules in a cylinder= $\pi d^2 vn$

No. of collision in a second= $\pi d^2 vn$

Average time interval between two successive collision= $\frac{1}{\pi d^2 vn}$ sec

Average distance between two successive collision= $\frac{1}{\pi d^2 vn}$ x v = $\frac{1}{\pi d^2 n}$



Expression for mean free path:

The mean free path (λ) = $\frac{1}{\pi d^2 n}$ *(i)*

$\pi d^2 = \sigma$ *is microscopic collision cross section of molecule*

$\lambda = \frac{1}{\sigma n}$ *(ii)*

Claussius expression

$\lambda = \frac{3}{4\pi d^2 n}$ *(iii)*

Maxwell's expression

$\lambda = \frac{1}{\sqrt{2}\ \pi d^2 n}$ *(iv)*

From eqn I to iv $\lambda \propto \frac{1}{d^2}$ *if m is mass of each molecule density* $\rho = mn$

$\lambda = \frac{1}{\sqrt{2}} \frac{m}{\pi d^2 \rho}$ *ie* $\lambda \propto \frac{1}{\rho}$ *If P is pressure then P =nkT* $\therefore n = \frac{P}{kT}$

$$\lambda = \frac{1}{\sqrt{2}} \frac{kT}{\pi d^2 P}$$

$$\lambda \propto T$$

Expression for mean free path:

CONCLUSION:

1. *Smaller the size of molecule , larger mean free path* ($\lambda \propto \frac{1}{d^2}$)
2. *Increase in temperature ,increases separation of molecules and causes increase in mean free path*($\lambda \propto T$)
3. *Decrease in density means molecules are widely separated resulted in increase in mean free path*
4. *Reduction of pressure of gas ,increases the molecular distance hence mean free path increases* ($\lambda \propto \frac{1}{P}$)

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Transport Phenomena:
Viscosity: Transport of momentum

Different layers of gas may have different velocities

Relative motion of layers of gas wrt other

To be steady state , layers moving faster will transfer momentum to slower

This phenomena is viscosity of gas

Phenomena of viscosity of gas is due to transfer of momentum

Viscosity is property of fluid

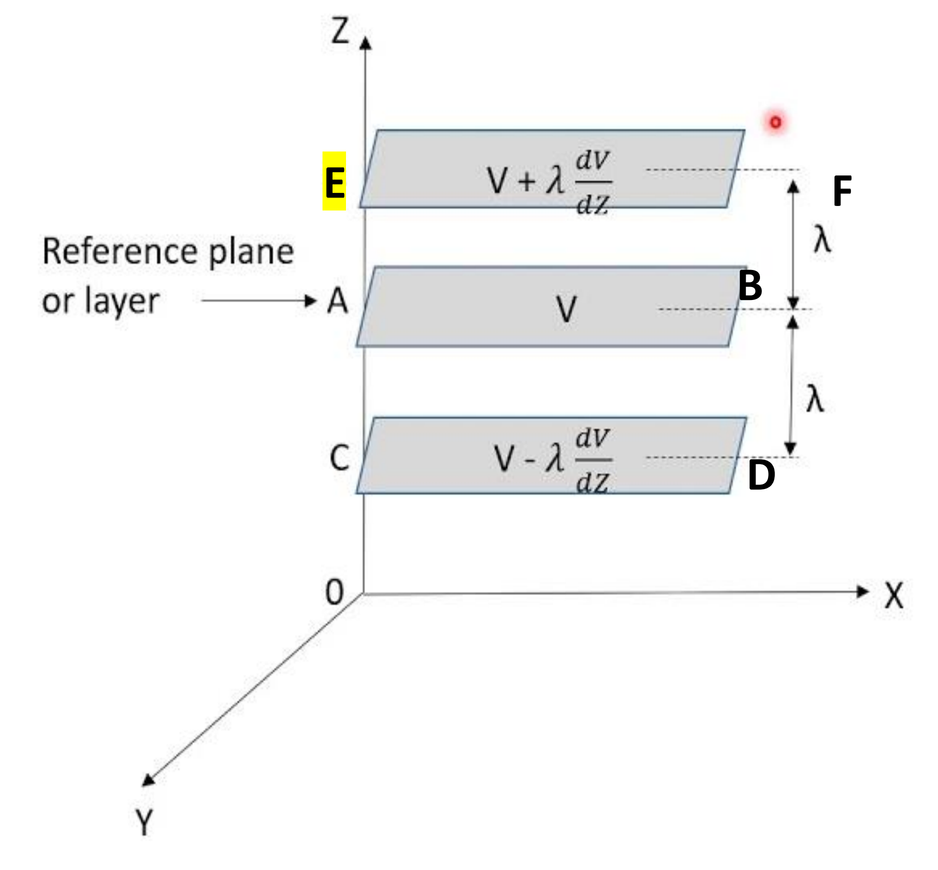
Occurs due to frictional forces between adjacent layers moving parallel to each others

Definition :

Tangential force per unit area required to maintain unit velocity gradient

Transport Phenomena:

Viscosity: Transport of momentum



Newtons equation for viscosity is

$$F = -\eta\, A\, \frac{dv}{dz} \qquad \text{------(1)}$$

Suppose gas flows over horizontal surface OX

Velocity of layer in contact with OX is zero and gradually increases along OZ

The change is unform with rate dv/dz

Consider layer AB at certain distance from OX

Velocity of layer AB is v

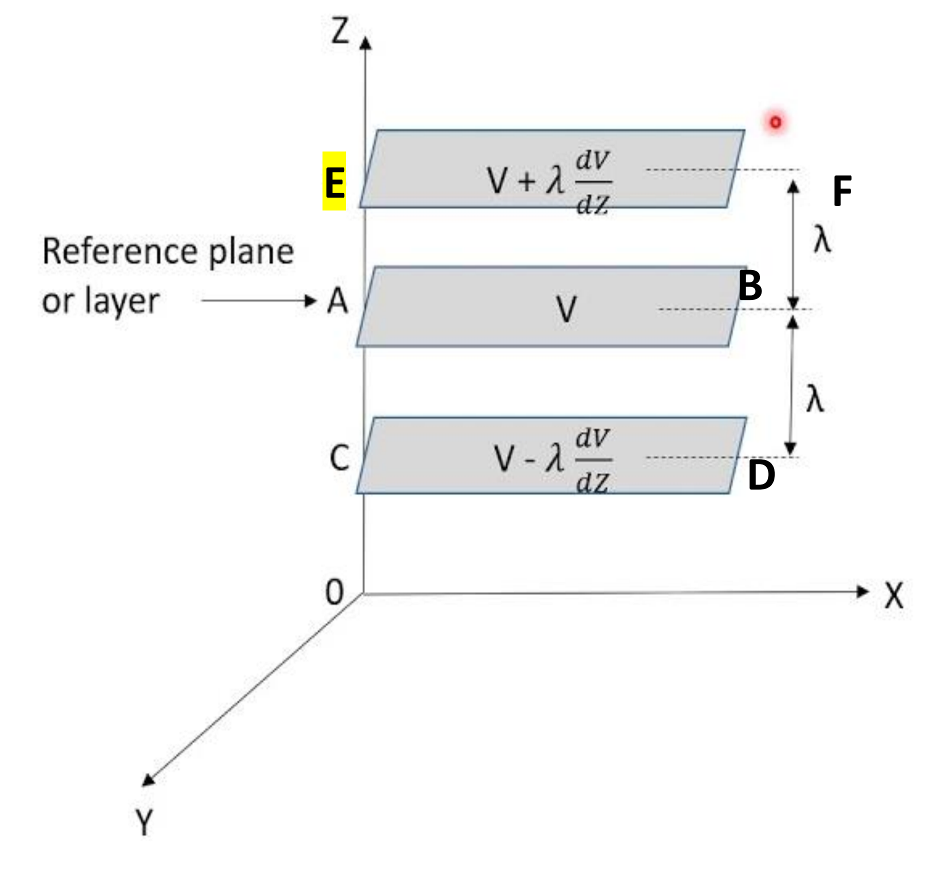
Consider two layers EF and CD above and below AB respectively at a distance mean free path λ

Velocity of gas in layer EF = $v + \frac{dv}{dz}\lambda$

Velocity of gas in layer CD = $v - \frac{dv}{dz}\lambda$

Transport Phenomena:

Viscosity: Transport of momentum



As molecules are moving in all possible direction parallel to X,Y, and Z axes

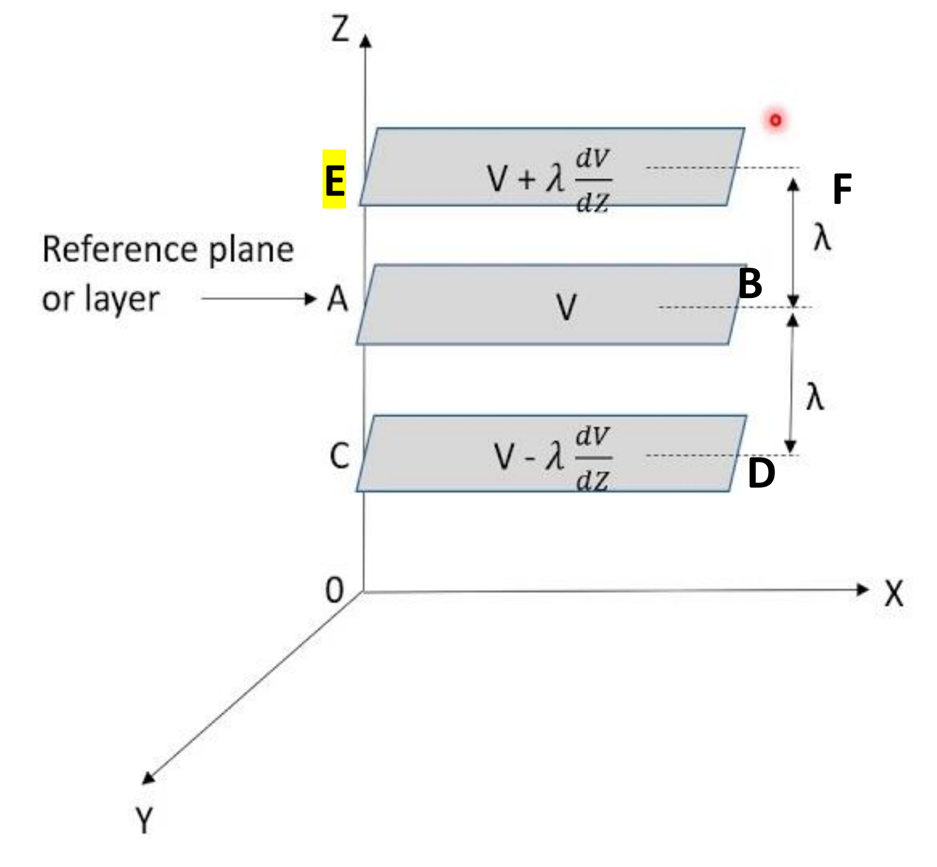
1/6 molecules moving parallel to any one axis in one particular direction

m is mass of each molecule and n is number of molecules per cc and c is average speed

Number of molecules passing downward from EF to CD per unit area of layer AB in one second= $\frac{nc}{6}$

Momentum carried downward from EF to CD per unit area of layer AB in one second=$\frac{mnc}{6}\left(v + \frac{dv}{dz}\lambda\right)$

Transport Phenomena:
Viscosity: Transport of momentum



Similarly number of molecules passing upward from CD to EF per unit area of layer AB in one second= $\frac{nc}{6}$

Momentum carried upward from CD to EF per unit area of layer AB in one second= $\frac{mnc}{6}\left(v - \frac{dv}{dz}\lambda\right)$

Net momentum lost by layer EF and gain by CD per unit area of layer AB in one second

$= \frac{mnc}{6}\left(v + \frac{dv}{dz}\lambda\right) - \frac{mnc}{6}\left(v - \frac{dv}{dz}\lambda\right)$

$= \frac{mnc}{6}\left\{\left(v + \frac{dv}{dz}\lambda\right) - \left(v - \frac{dv}{dz}\lambda\right)\right\}$

$= \frac{1}{3} mnc\lambda \frac{dv}{dz}$ this is rate of change of momentum

Transport Phenomena:
Viscosity: Transport of momentum

$= \frac{1}{3} mnc\lambda \frac{dv}{dz}$ this is rate of change of momentum

According to Newton's second law of motion

Rate of change of momentum is force

$F = \frac{1}{3} mnc\lambda \frac{dv}{dz}$ ---(2)

From equation (1) and (2)

$$\eta A \frac{dv}{dz} = \frac{1}{3} mnc\lambda \frac{dv}{dz}$$

For unit area A=1

$\therefore \eta = \frac{1}{3} mnc\lambda = \frac{1}{3} \rho c\lambda$ since $mn=\rho$

Transport Phenomena:
Viscosity: Transport of momentum

$$\eta = \frac{1}{3}\ mnc\lambda$$

But $\lambda = \frac{1}{\sqrt{2}\ \ \pi d^2 n}$

$$\eta = \frac{1}{3}\ mnc\frac{1}{\sqrt{2}\ \ \pi d^2 n}$$

$$\eta = \frac{mc}{3\sqrt{2}\ \ \pi d^2}$$

As $c \propto \sqrt{T}$, $\eta \propto \sqrt{T}$

$\rho \propto p$ and $\lambda \propto \frac{1}{P}$, then $\rho\lambda = constant$ η is independent of pressure

Transport Phenomena:
Viscosity: Transport of momentum

CONCLUSION:

1. *Viscosity phenomena is due to transport of momentum*

2. *With increase of temperature, thermal agitation increases, velocity in a direction also increases resulting in increase of viscosity with temperature ($\eta \propto \sqrt{T}$)*

1. *At sufficiently high pressure coefficient of viscosity of gas is independent of pressure*

Transport Phenomena:
Thermal conductivity of gases: Transport of Energy

Different layers of gas may have different temperatures

The molecules at higher temperature will have greater energy

To be steady state , molecules will transfer energy from region of higher temperature to lower temperature

This phenomena is thermal conductivity of gases

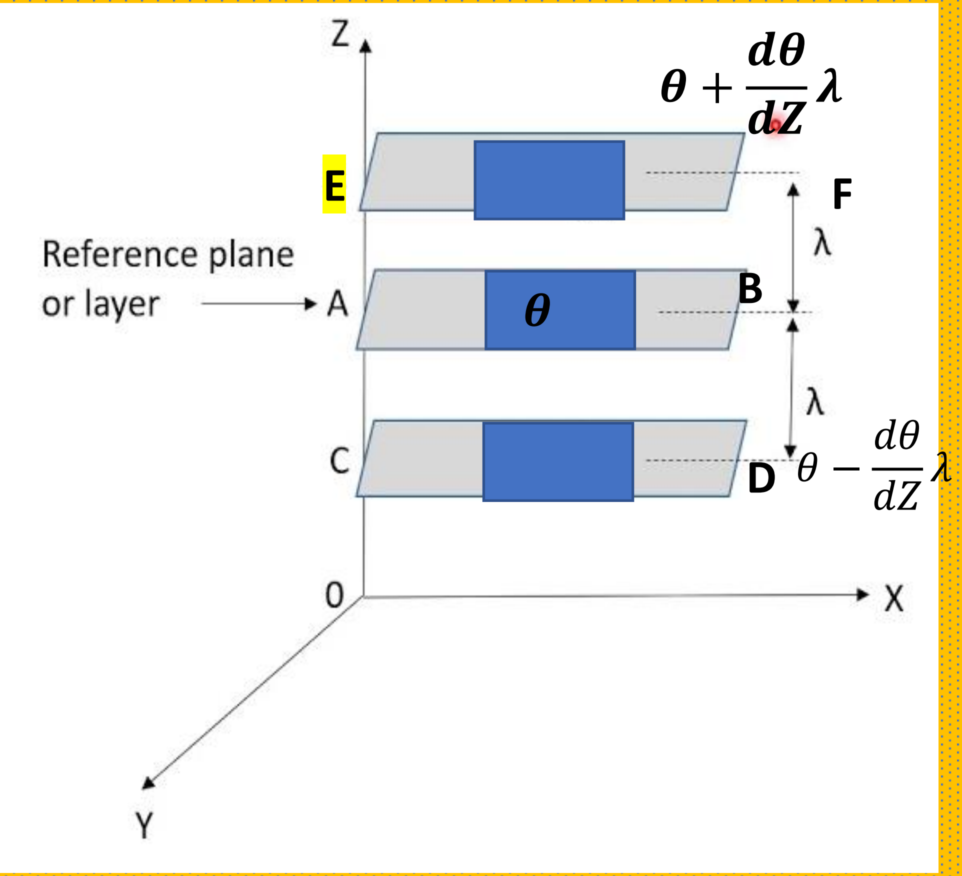
Phenomena of thermal conductivity of gases is due to transfer of energy

Thermal conductivity occurs when temperature gradient exists

Definition :

Amount of heat conducted per unit time per unit area per unit temperature gradient when material is in steady state

Transport Phenomena:
Thermal conductivity of gases:
Transport of Energy



Equation of thermal conductivity of gas is

$Q = K\ A\left(\frac{d\theta}{dz}\right)t$ ------(1)

Suppose gas flows over horizontal surface OX

The layer in contact with OX is cold and and temperature gradually increases along OZ

The change is unform with rate $d\theta/dz$

Consider layer AB at certain distance from OX

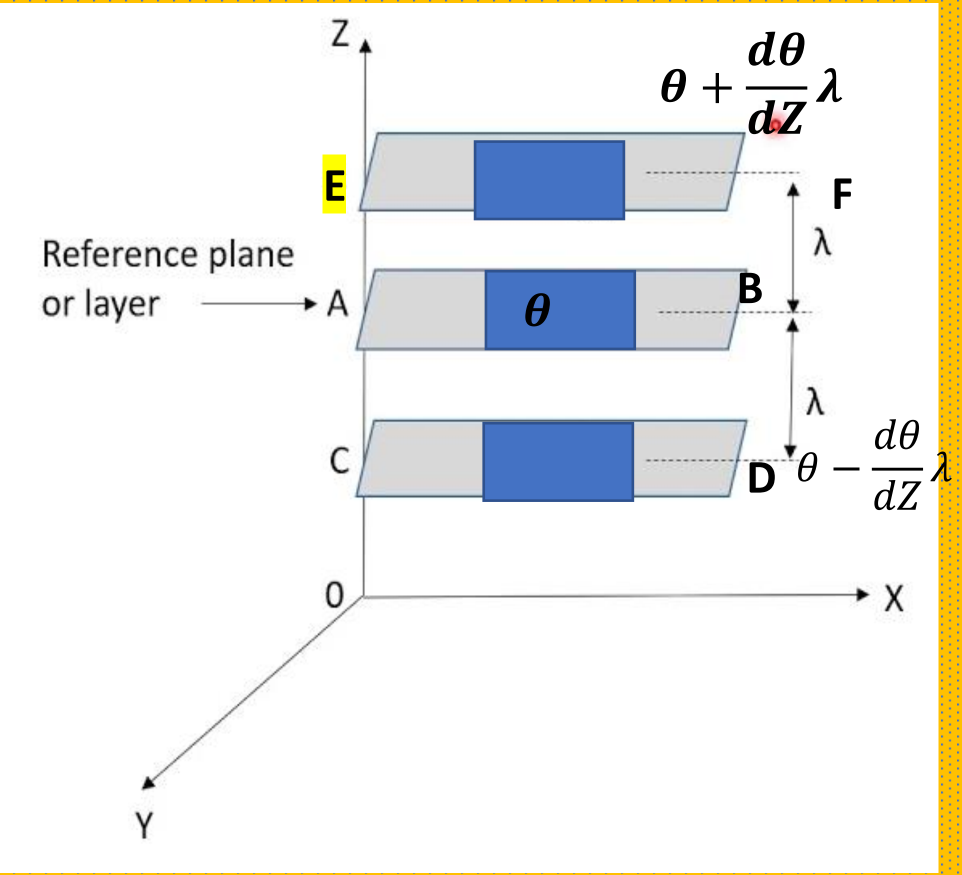
Temperature of layer AB is θ

Consider two layers EF and CD above and below AB respectively at a distance mean free path λ

Temperature of gas in layer EF $= \theta + \frac{d\theta}{dZ}\lambda$

Temperature of gas in layer CD $= \theta - \frac{d\theta}{dz}\lambda$

Transport Phenomena:
Thermal conductivity of gases:
Transport of Energy



As molecules are moving in all possible direction parallel to X,Y, and Z axes

1/6 molecules moving parallel to any one axis in one particular direction

m is mass of each molecule and n is number of molecules per cc and c is average speed c_v is specific heats of gas

Energy of molecule of gas $Q = mc_v\theta$

Number of molecules passing downward from EF to CD per unit area of layer AB in one second= $\frac{nc}{6}$

Energy carried downward from EF to CD per unit area of layer AB in one second=$\frac{mncc_v}{6}\left(\theta + \frac{d\theta}{dz}\lambda\right)$

Transport Phenomena:
Thermal conductivity of gases:
Transport of Energy

Similarly number of molecules passing upward from CD to EF per unit area of layer AB in one second= $\frac{nc}{6}$

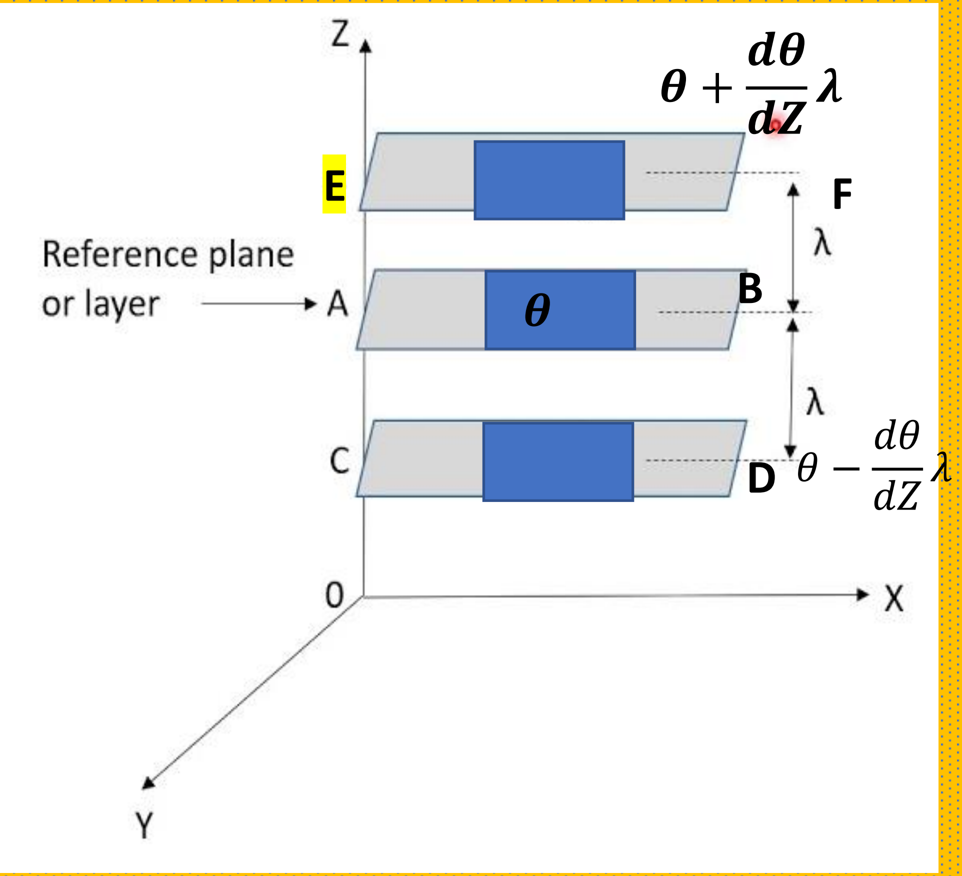
Energy carried upward from CD to EF per unit area of layer AB in one second=$\frac{mncc_V}{6}\left(\theta-\frac{d\theta}{dz}\lambda\right)$

Net energy lost by layer EF and gain by CD per unit area of layer AB in one second

=$\frac{mncc_V}{6}\left(\theta+\frac{d\theta}{dz}\lambda\right)-\frac{mncc_V}{6}\left(\theta-\frac{d\theta}{dz}\lambda\right)$

$$=\frac{mncc_V}{6}\left\{\left(\theta+\frac{d\theta}{dz}\lambda\right)-\left(\theta-\frac{d\theta}{dz}\lambda\right)\right\}$$

$=\frac{mncc_V\lambda}{6}\frac{dv}{dz}$ this is rate of change of energy



Transport Phenomena:
Thermal conductivity of gases: Transport of Energy

$= \frac{mncc_V\lambda}{3} \frac{d\theta}{dz} \quad ---(2)$ this is rate of change of energy i.e. Heat

From equation (1) and (2)

$$Q = K\ A\left(\frac{d\theta}{dz}\right)t = \frac{mncc_V\lambda}{3} \frac{d\theta}{dz}$$

For unit area A=1

$\therefore K = \frac{1}{3} mncc_V\lambda \quad = \frac{1}{3}\ \rho cc_V\lambda$ since mn=ρ is density of gas

Transport Phenomena:
Thermal conductivity of gases: Transport of Energy

$$K = \frac{1}{3} mncc_V \lambda$$

But $\lambda = \frac{1}{\sqrt{2} \; \pi d^2 n}$

K$= \frac{1}{3} \; mncc_V \frac{1}{\sqrt{2} \; \pi d^2 n}$

K $= \frac{mcc_V}{3\sqrt{2} \; \pi d^2}$

As $c \propto \sqrt{T}$, K $\propto \sqrt{T}$

$\rho \propto p$ and $\lambda \propto \frac{1}{P}$, then $\rho\lambda = constant$ K is independent of pressure

Transport Phenomena:

Thermal conductivity of gases: Transport of Energy

$$K = \frac{1}{3}\, mncc_V \frac{1}{\sqrt{2}\;\; \pi d^2 n}$$

But $m = \frac{M}{N}$ i.e. $mass\ of\ molecule = \frac{Molecular\ weight}{Avagadro\ Number}$

And $c_v = \frac{C_v}{M}$ i.e. $Principal\ sp.heats\ of\ gas = \frac{Molar\ sp.heats\ of\ gas}{Molecular\ weight}$

$$\mathrm{K} = \frac{1}{3\sqrt{2}} \frac{c}{\pi d^2} \left(\frac{M}{N}\right) \frac{C_v}{M}$$

$$K = \frac{1}{3\sqrt{2}} \frac{C_v}{N\pi d^2} c$$

But average speed $c = \sqrt{\frac{8KT}{\pi m}}$ Thermal conductivity is inversely proportional to sq. root of mass of molecule

Transport Phenomena:
Thermal conductivity of gases: Transport of Energy

CONCLUSION:

1. *Thermal conductivity of gas phenomena is due to transport of energy*
2. *With increase of temperature, thermal agitation increases, resulting in increase of Thermal conductivity with temperature* ($K \propto \sqrt{T}$)
3. *At sufficiently high pressure coefficient of Thermal conductivity of gas is independent of pressure*
4. *Mass of hydrogen is least, the thermal conductivity of hydrogen will be larger*

Transport Phenomena:
Diffusion of gas: Transport of mass

Different layers of gas may have different concentration i.e.(number of molecules per cc

The molecules at higher concentration will transfer to lower concentration

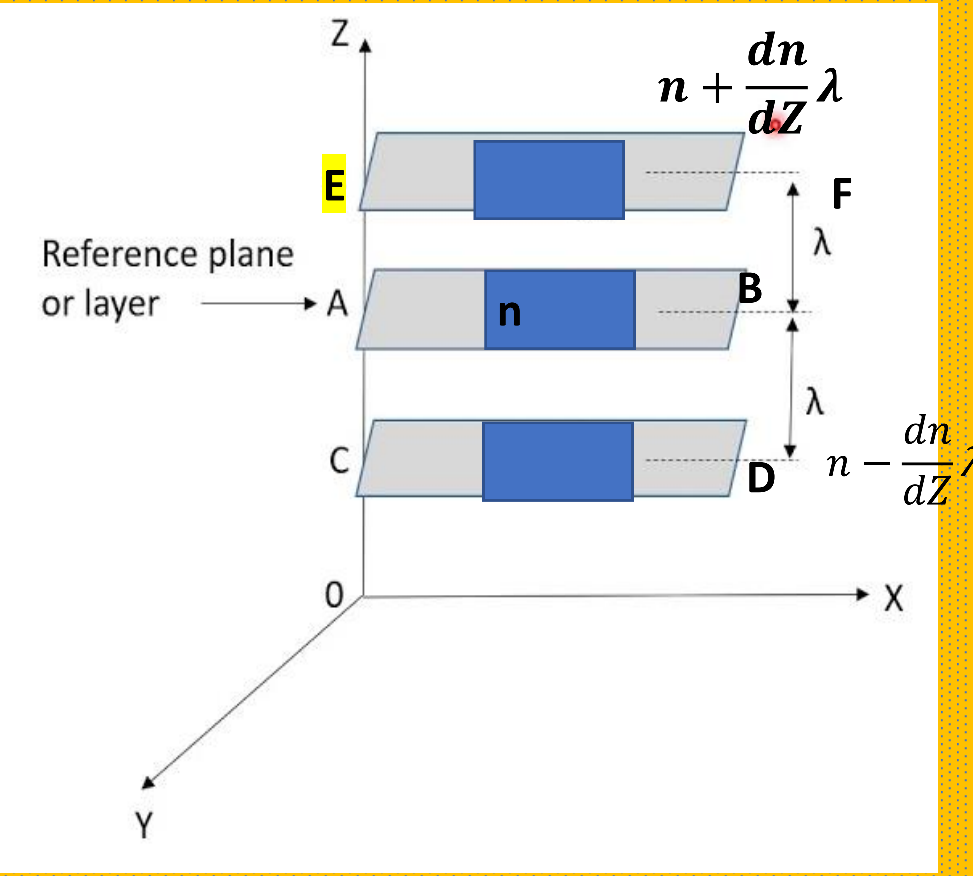
To be steady state , molecules diffuse from higher concentration region to lower concentration region

This phenomena is diffusion of gases

Phenomena of diffusion of gases is due to transfer of ***mass***

Coefficient of diffusion : is defined as ratio of number of molecules crossing per unit area in one second to the rate of change of concentration with distance

Transport Phenomena:
Diffusion of gas: Transport of mass



Equation of coefficient of diffusion of gas is

$$D = \frac{dN/dt}{dn/dz} \text{------(1)}$$

Suppose gas flows over horizontal surface OX

The layer in contact with OX is at lower concentration and concentration gradually increases along OZ

The change is unform with rate dn/dz

Consider layer AB at certain distance from OX

Concentration of layer AB is n

Consider two layers EF and CD above and below AB respectively at a distance mean free path λ

Transport Phenomena:
Diffusion of gas: Transport of mass

Concentration of gas in layer EF $= n + \frac{dn}{dZ}\lambda$

Concentration of gas in layer CD $= n - \frac{dn}{dz}\lambda$

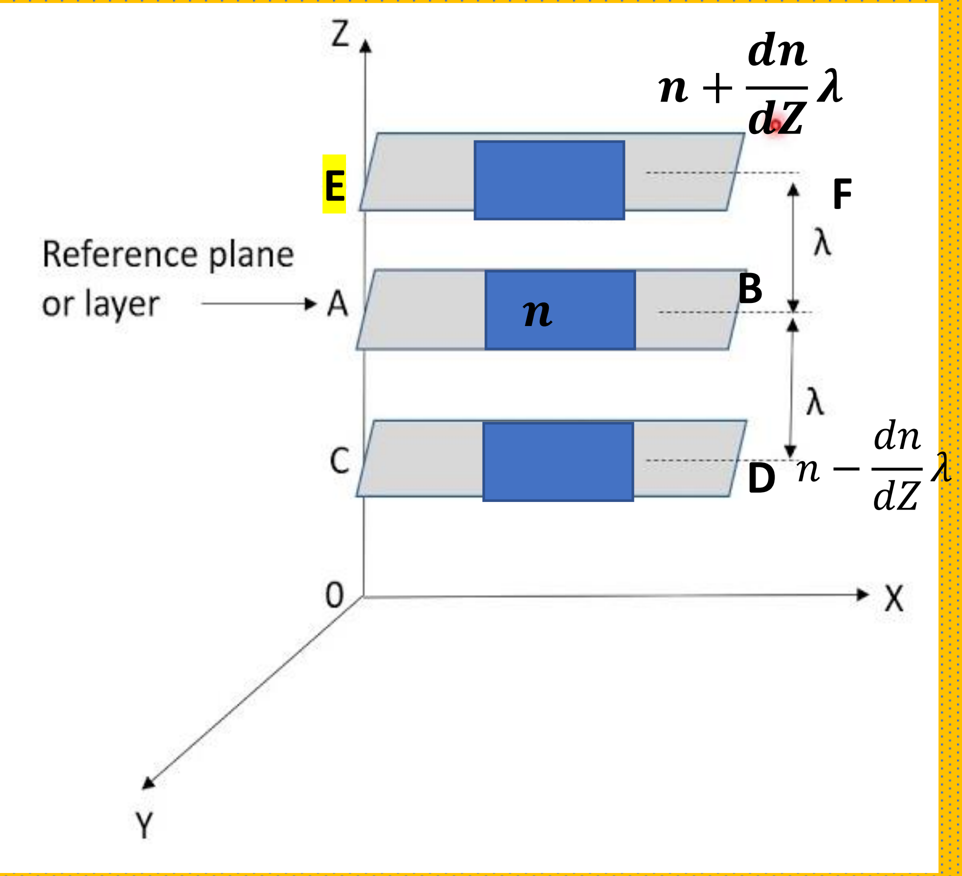
As molecules are moving in all possible direction parallel to X,Y, and Z axes

1/6 molecules moving parallel to any one axis in one particular direction

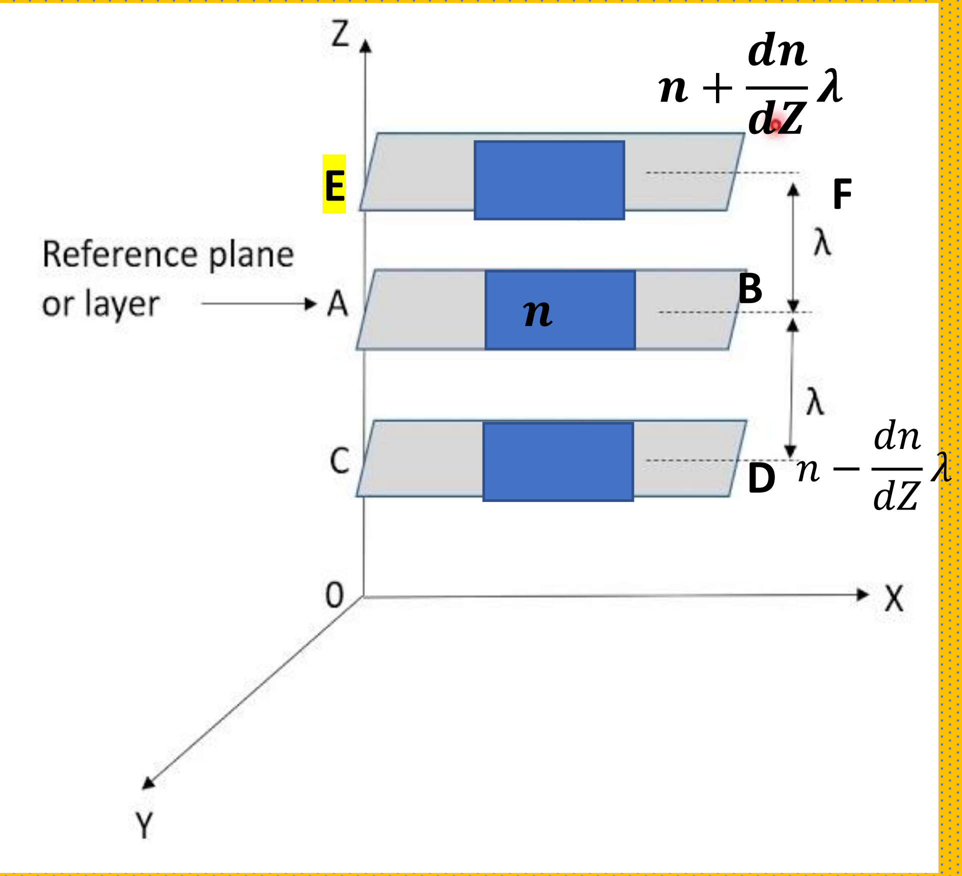
If c is average speed of molecules

Number of molecules passing downward from EF to CD per unit area of layer AB in one second

$= \frac{c}{6}\left(n + \frac{dn}{dz}\lambda\right)$



Transport Phenomena:
Diffusion of gas: Transport of mass



Similarly number of molecules passing upward from CD to EF per unit area of layer AB in one second

$$=\frac{c}{6}\left(n-\frac{dn}{dz}\lambda\right)$$

Net number of molecules lost by layer EF and gain by CD per unit area of layer AB in one second

$$=\frac{c}{6}\left(n+\frac{dn}{dz}\lambda\right)-\frac{c}{6}\left(n-\frac{dn}{dz}\lambda\right)$$

$$=\left\{\frac{c}{6}\left(n+\frac{dn}{dz}\lambda\right)-\left(n-\frac{dn}{dz}\lambda\right)\right\}$$

$$=\frac{1}{3}\mathrm{c}\lambda\frac{dn}{dz}$$ Gives rise to phenomenon of diffusion

Transport Phenomena:
Diffusion of gas: Transport of mass

From equation (1) and (2)

$$D = \frac{dN/_{dt}}{dn/_{dz}} = \frac{1}{3}c\lambda\frac{dn}{dz}$$

$$\therefore D = \frac{1}{3}c\lambda$$

As $c \propto \sqrt{T}$, And $\lambda \propto \frac{T}{P}$ D $\propto T^{3/2}$

Transport Phenomena:
Diffusion of gas: Transport of mass

CONCLUSION:

1. *Diffusion of gas phenomena is due to transport of* ***mass***

2. *With increase of temperature, rate of flow of molecules increases* ($D \propto T^{3/2}$)

3. *At low pressure , rate of flow of molecules decreases* ($D \propto \frac{1}{P}$)

Inter Relation Between Three Transport Coefficients:

Coefficient of viscosity of gas $\eta = \frac{1}{3}\rho c\lambda$ ------(1)

Coefficient of thermal conductivity of gas $K = \frac{1}{3}\rho c c_V\lambda$----(2)

Coefficient of Diffusion of gas $D = \frac{1}{3}c\lambda$ -------(3)

From equation 1 and 2

$$\frac{\eta}{K} = \frac{1}{c_v} \quad OR \quad \eta = \frac{K}{c_v} \quad OR \quad K = \eta c_v$$

From equation (2) and (3)

$$\frac{D}{K} = \frac{1}{\rho c_v} \quad OR \quad D = \frac{K}{\rho c_v} \quad OR \quad K = D\rho c_v$$

From equation (1) and (3)

$$\frac{\eta}{D} = \rho \quad OR \; \eta = D\rho \quad OR \quad D = \frac{\eta}{\rho}$$